152:5–7 & 9–11 **2/15/2007**

Some review questions from *me* for the first exam

You should *expect* to be asked about the following topics:

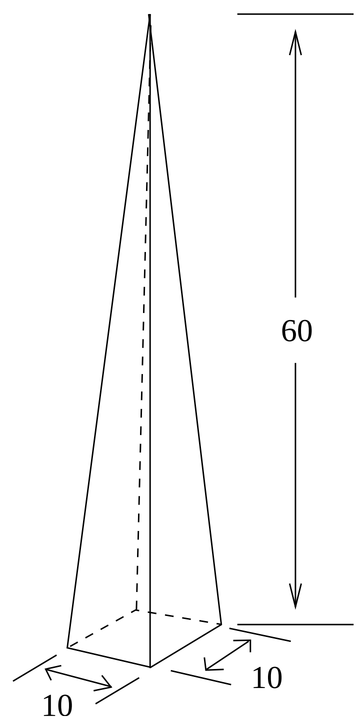
• Definite integrals evaluating volumes (volumes with slicing symmetry, volumes of revolution), average values, and work • Methods of antidifferentiation (substitution, integration by parts, integrals of trig functions, trig substitutions, and partial fractions) • Strategies for numerical (approximate) integration • Improper integrals (taking limits of "proper" integrals; concluding finiteness by comparing to known, simpler integrals).

Below are questions which I have given on exams in the past about this material to second-semester calculus classes. These are questions from *more than one exam*. Your exam will be shorter!

1. Suppose $\mathcal{R}$ is the region bounded by $y = \frac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$.

a) Find the volume of the solid that results from rotating $\mathcal{R}$ around the x-axis.

b) Find the volume of the solid that results from rotating $\mathcal{R}$ around the y-axis.



2. A flat-sided monolith* is 60 feet tall with a square base that is 10 feet on each side. What is the volume of the monolith?

3. a) Suppose w is a positive number. Define $A(w)$ to be the average value of $(\cos x)^2$ on the interval $0 \leq x \leq w$. Compute $A(w)$, and show how the integral is calculated.

b) What is the limit of $A(w)$ as $w \to 0^+$?

4. Use the method of partial fractions to verify that $\int_0^1 \frac{1}{(x+1)(x^2+1)}\,dx = \frac{1}{4}\ln 2 + \frac{1}{8}\pi$.

5. a) Here's a formula from the *Tables of Indefinite Integrals* by G. Petit Bois (1906): $\int \frac{x^2}{x^3+5x^2+8x+4}\,dx = \log(x+1) + \frac{4}{x+2}$.
Please verify this formula using the method of partial fractions.

b) Here's another formula from the same text: $\int \frac{x^2}{x^3+x^2+x+1}\,dx = \frac{1}{2}\log\left((x+1)\sqrt{x^2+1}\right) - \frac{1}{2}\arctan x$.
Again, please verify this formula using the method of partial fractions.

6. Calculate the following integrals, showing your work.

a) $\int \frac{dx}{\sqrt{1+x^2}}$

b) $\int \frac{dx}{2e^x+1}$

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7. a) Explain why $\int_1^{\infty} \frac{1}{x^2+e^{2x}}\, dx$ converges.

b) Explain why the value of the integral in a) is less than $\frac{1}{2}$.

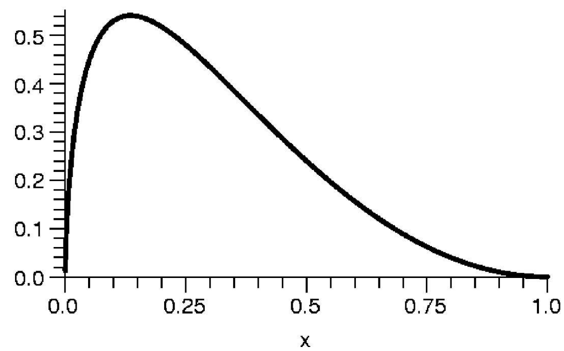
8. In this problem, $f(x) = x\,(\ln x)^2$.

a) Verify that $\lim\limits_{x\to 0^+} f(x) = 0$.

Hint Write the limit so you can apply L'H, but be sure to indicate *why* you need L'H *whenever* you use it.

b) Compute the improper integral $\int_0^1 f(x)\, dx$. Indicate why the limits you need exist, and what these limits are.

c) Here's a graph of $x(\ln x)^2$ drawn by `Maple` on the interval $[0, 1]$. Does this graph help to confirm your computation in b)? Why?

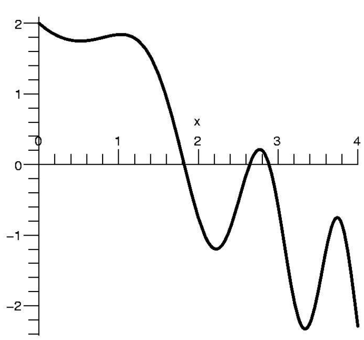


9. Use a substitution followed by integration by parts to verify that $\int_0^1 e^{\sqrt{x}}\, dx = 2$.

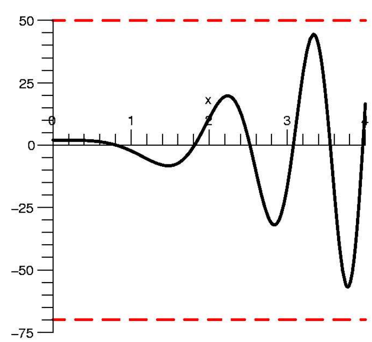
10. The integral $\int_0^2 \left(x^3+1\right)^{7/2}\, dx$ is approximated using the Trapezoidal Rule by dividing $[0, 2]$ into n segments of equal length. How large should n be in order to guarantee that the error is at most 10^{-6}?

Note You must give some reason explaining why any overestimates of derivatives you make are valid on the entire interval.

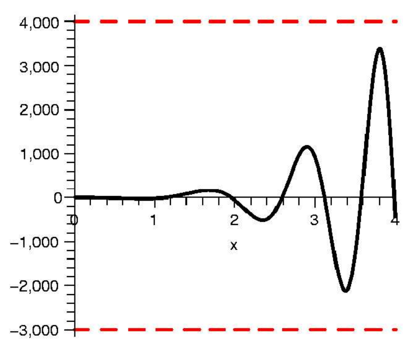
11. In this problem, $f(x) = 2 - x + \sin(x^2)$. Assume that the graphs of the functions below on the interval $[0, 4]$ are correct. Information from these graphs may be used to answer the questions which follow.



The graph of $y = f(x)$



The graph of $y = f''(x)$, the second derivative of f



The graph of $y = f^{(4)}(x)$, the fourth derivative of f

a) How many subdivisions are needed to estimate $\int_0^4 f(x)\, dx$ with the Trapezoidal Rule to an accuracy of 10^{-10}? **Suggestion** Use the picture and the formula sheet.

b) How many subdivisions are needed to estimate $\int_0^4 f(x)\, dx$ with Simpson's Rule to an accuracy of 10^{-10}? **Suggestion** Use the picture and the formula sheet.