Some review questions from *me* for the second exam

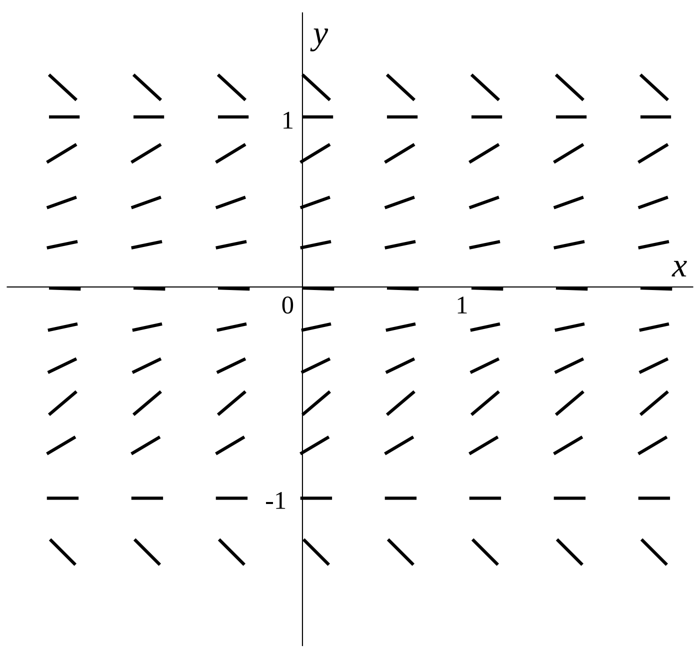
You should *expect* to be asked about the following topics:

- • Solutions to differential equations (exact solutions to separable equations; qualitative information using direction fields) • Basic convergence questions about sequences and series • Series convergence using the Integral Test, the Comparison Test, the Limit Comparison Test, the Alternating Series Test, the Ratio Test, and the Root Test • Power series (interval of convergence and radius of convergence) and simple manipulations of power series which can be obtained from geometric series.

Below are questions which I have given on exams in the past about this material to second-semester calculus classes. These are questions from *more than one exam.* Your exam will be shorter!

1. Find the solution of the differential equation $\frac{dy}{dx} = \frac{xy^3}{x^2+1}$ satisfying the initial condition $y(0) = 3$. In the answer express y explicitly as a function of x.

2. Below is part of the direction field for the differential equation $y' = y^2(1-y)(1+y)$.



a) Find all numbers k so that the constant function $f(x) = k$ is a solution of this differential equation (these are the *equilibrium solutions*).

b) Sketch a typical solution curve $y = f(x)$ to this differential equation when $0 < y(0) < 1$.

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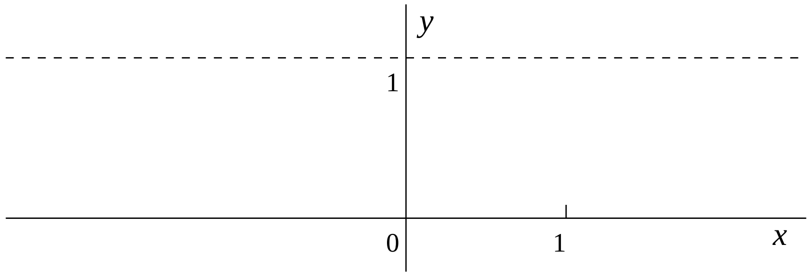


What is $\lim_{x\to+\infty} f(x)$? __________ What is $\lim_{x\to-\infty} f(x)$? __________

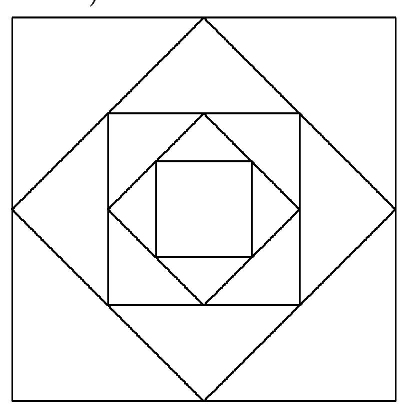
3. Find the interval of convergence and the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n x^n}{\sqrt{n}}$. In addition, determine whether the series is absolutely or conditionally convergent at the boundary points of the interval of convergence.

4. The series $\sum_{n=1}^{\infty} \frac{1}{3n^2+5n+7}$ converges. Find a specific finite sum of rational numbers (quotients of integers) which is within .0001 of the sum of the infinite series. Be sure to explain why your error estimate is correct.
Hint Compare the "infinite tail" to something simpler, and analyze that.

5. a) Suppose the sequence $\{A_n\}$ is defined by $A_n = \frac{6\cdot 4^n+7n^3}{5\cdot 4^n+8n^2}$. What is $\lim_{n\to\infty} A_n$?
b) Suppose the sequence $\{B_n\}$ is defined by $B_n = \left(1+\frac{3}{n}\right)^{5n}$. What is $\lim_{n\to\infty} B_n$?
Hint ln and l'H.

6. a) What is the Taylor series for $f(x) = 3 - 11x^5$ centered at $a = 0$? Why?
b) Explain briefly why $g(x) = |x|$ has no power series (Taylor series) centered at $a = 0$.

7. An infinite sequence of squares is drawn (the first five are shown), with the midpoints of the sides of one being the vertices of the next. The outermost square has sides which are 1 unit long. What is the <u>sum</u> of the <u>perimeters</u> of all of the squares?



8. Determine whether each of the following sequences converges, and find the limits when they exist. Explain your answers.
a) $a_n = \frac{\ln(\ln(n^4))}{(\ln n)^3}$ b) $b_n = \sqrt[n]{6n^8}$

9. Determine whether each of the following infinite series converges or diverges. Explain your answers.
a) $\sum_{n=1}^{\infty} \frac{1}{8n^6 - \frac{1}{n^2}}$ b) $\sum_{n=1}^{\infty} \frac{(3n)!}{(n!)^2(2n)!}$.

10. Find power series centered at 0 (called the Taylor series or the Maclaurin series) for the following functions:
a) $\frac{1}{1+x^3}$ b) $\frac{5}{x-3}$ c) $\frac{\ln(1+x^2)}{x}$