

NAME _____

SECTION _____

There are two problems.

1. The integral $\int_0^2 \frac{1}{\sqrt{2-x}} dx$ is an improper integral.

You must analyze this improper integral using the following sequence of steps:

First Write this integral as a limit of proper definite integrals with a varying parameter.

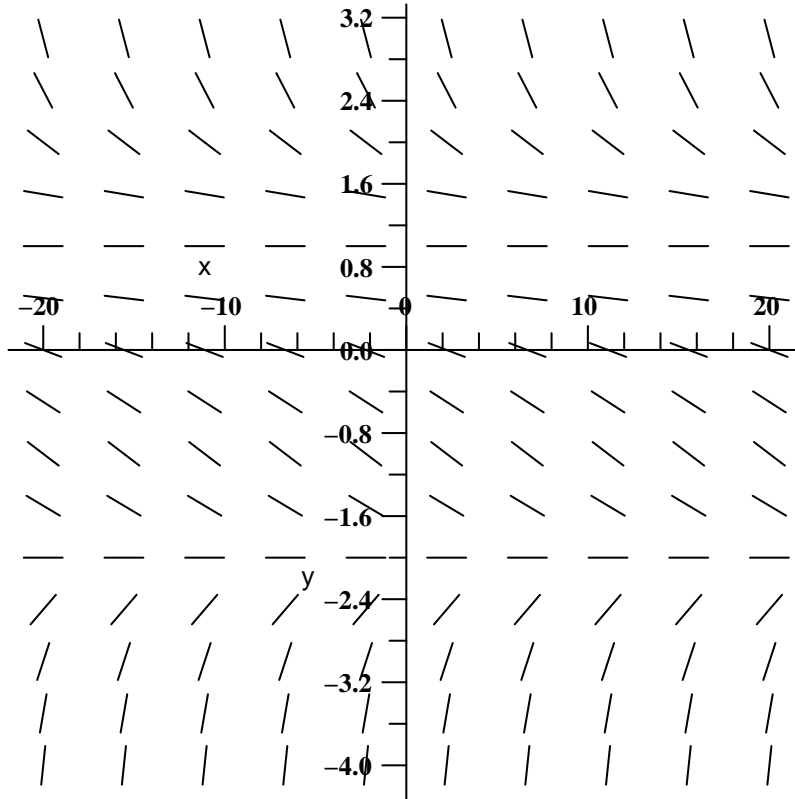
Second Evaluate the definite integral with a parameter which appears inside the limit (neither the word “limit” nor the term \lim_{\rightarrow} should appear in this stage). Your answer should include one or more expressions with the parameter.

Third Use the previously computed answer and the limit expression you got in the first part of this problem to decide if the improper integral converges, and, if it does, find the value of the integral.

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2. The horizontal and vertical axes on the graph below have different scales. The graph is a direction field for the differential equation

$$y' = -\frac{1}{30}(y-1)^2(2+y).$$



a) Find the equilibrium solutions (where y doesn't change) for this differential equation.

b) Sketch solution curves on the axis above through these points. Find the indicated limits:

- $(0, 0)$. Label this curve **A**. On curve **A**, $\lim_{x \rightarrow \infty} y(x) = \underline{\hspace{2cm}}$.
- $(0, 2)$. Label this curve **B**. On curve **B**, $\lim_{x \rightarrow \infty} y(x) = \underline{\hspace{2cm}}$.
- $(0, -3)$. Label this curve **C**. On curve **C**, $\lim_{x \rightarrow \infty} y(x) = \underline{\hspace{2cm}}$.
- $(1, 0)$. Label this curve **D**. On curve **D**, $\lim_{x \rightarrow \infty} y(x) = \underline{\hspace{2cm}}$.

c) One of the equilibrium solutions is a *stable* equilibrium (nearby solutions approach this solution as $x \rightarrow +\infty$). Which equilibrium solution is the stable solution?