Math 152, Spring 2007: Review Questions for Exam 1

What's below are review questions prepared by the course coordinator for the first exam. Problems 2 and 11 need not be considered for students in these sections, since that material will not be on our first exam.

You should not assume that the first midterm exam will resemble this review sheet. The problems below were chosen because they seemed interesting or because they illustrated an important concept. They are not meant to be typical exam problems.

(1) Assume a, b, c, d are real numbers such that $0 < a < b$, $0 < c < d$. The inequalities $a \le x \le b$, $c \le y \le d$ determine a rectangle in the xy-plane. We get a solid by rotating this rectangle about the y-axis. Find the volume of this solid using two methods: The method of shells and the method of washers. Make sure that the two methods give you the same answer.

(2) Use calculus to find the surface area of a sphere of radius R.

(3) A heavy chain with a density of 3 pounds per foot is stored on a shelf that is placed 4 feet above the floor. A passing truck produces vibrations in the shelf which rattle the chain and cause it to unravel. A 10 foot section of chain cascades over the edge of the shelf. How much work (in foot-pounds) is required to put this portion of the chain back on the shelf? Note that 6 feet of chain are *on* the floor, not *under* the floor.

(4) Evaluate $\int (\cos^3 x)(\sin^3 x)\,dx$ in two different ways that do not involve reduction formulas. Verify that your two answers are actually the same (even though they may seem to be very different).

(5) Evaluate $\int (\tan^5 x)(\sec^6 x)\,dx$ in two different ways that do not involve reduction formulas. Do not verify that your two answers are actually the same.

(6) Evaluate $\int \cos(3x)\cos(5x)\,dx$ using two integrations by parts.

(7) Evaluate $\int \sec x\,dx$ using the fact $\sec x = \dfrac{\cos x}{\cos^2 x}$, a trigonometric identity and partial fractions. Verify that your answer is equivalent to the usual answer.

(8) Evaluate $\displaystyle\int \frac{1}{1+e^x}\,dx$ in two ways: First, use the fact $1 = (1+e^x) - e^x$. Second, multiply and divide by e^x. Check that you get the same answer.

(9) An inefficient way to approximate $\ln 3$ is to use Simpson's Rule with n subintervals applied to $\displaystyle\int_1^3 \frac{dx}{x}$. What values of n will guarantee an accuracy better than 10^{-4}?

(10) Evaluate $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$.

(11) Solve the differential equation $\dfrac{dy}{dx} = -xy^3$ with initial condition $y(0) = 1$.

(12) Evaluate $\displaystyle\int \frac{x^3+2x^2+7x+9}{(x^2+4)^2}\,dx$.