Math 152, Spring 2007: Review Questions for Exam 2

Below are review questions prepared by the course coordinator for the second exam. Problems 9 and 10 need not be considered now for students in these sections, since that material will not be on our second exam.

You should not assume that the second midterm exam will resemble this review sheet.

(1) Find the sums $\sum_{n=2}^{\infty}(4^{-n}+3^{-2n})$ and $\sum_{n=5}^{\infty}\frac{1}{(n-2)(n-1)}$.

(2) Assume that the numbers a_n are defined recursively by $a_1=1$, $a_{n+1}=\sqrt{7+a_n}$ for $n=1,2,3,\ldots$. Assume also that we have already shown that the sequence $a_1,a_2,a_3,\ldots$ converges. Find $\lim_{n\to\infty} a_n$.

(3) For each series below, determine whether it converges or diverges:

$$\sum_{n=2}^{\infty}\frac{n^n}{(n+2)!4^n},\quad \sum_{n=10}^{\infty}\frac{1}{n(\ln n)\sqrt{\ln(\ln n)}},\quad \sum_{n=3}^{\infty}\frac{4^n+\sqrt{n}}{5^n-n},\quad \sum_{n=4}^{\infty}\frac{(2n+1)!}{5^{n+3}(n+1)!n!}.$$

(4) Find the interval of convergence for each of the following power series:

$$\sum_{n=8}^{\infty}\frac{3^n(x+1)^n}{\ln n},\quad \sum_{n=1}^{\infty}\frac{(x-3)^{2n}}{\sqrt{n}},\quad \sum_{n=2}^{\infty}\frac{(-4)^n(x+2)^n}{n},\quad \sum_{n=2}^{\infty}\frac{(x-2)^n}{(n-1)!}.$$

(5) Find the first 3 nonzero terms of the Maclaurin series for $f(x)=\sec x$. There is no need to compute $f^{(n)}(x)$.

(6) Questions (a), (b), (c), (d) should be answered in the given order. (a) What is the Maclaurin series for $\frac{1}{1-x}$? (b) What is the Maclaurin series for $\frac{1}{(1-x)^2}$? (c) What is the Maclaurin series for $\frac{x}{(1-x)^2}$? (d) What is the sum of the series $\sum_{n=1}^{\infty}\frac{n}{5^n}$?

(7) For each series below, determine whether it converges or diverges:

$$\sum_{n=2}^{\infty}\frac{2+\sin(e^n+n^7)}{n+1+\cos(n^3-10)},\quad \sum_{n=1}^{\infty}\frac{3^n}{(n^4+20n^3+5)2^{n+4}},\quad \sum_{n=3}^{\infty}\frac{n^2\cos(\sqrt{n})}{\sqrt{n^7+n+3}}.$$

(8) How many terms of $\sum_{n=1}^{\infty}\frac{1}{n^3}$ should we use in order to approximate $\sum_{n=1}^{\infty}\frac{1}{n^3}$ with an accuracy better than 10^{-4}? How many terms of $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ should we use if we want to approximate $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ with an accuracy better than 10^{-4}?

(9) Assume that e^x is approximated by $\sum_{n=0}^{N}\frac{x^n}{n!}$ for $-0.1\le x\le 0.1$. Find a value of N that will give us an accuracy better than 10^{-8}.

(10) Find $\lim_{x\to 0}\frac{(e^x-1-x)^2}{1-x^2/2-\cos x}$. Do not use l'Hôpital's Rule.

(11) A bacterial population doubles every 10 days. How long does it take for the population to triple in size?

(12) Find the Maclaurin series for $f(x)=\int_0^x \tan^{-1}(t^5)\,dt$.

This problem from the first review collection is relevant to our second exam:

(11) Solve the differential equation $\frac{dy}{dx}=-xy^3$ with initial condition $y(0)=1$.