

1. Suppose that Q is the function defined by the formula

$$Q(x) = \left(\arctan(\ln(\sqrt{x} - 1)) \right)^3 .$$

a) What are the domain and range of Q ? Answers should *not* be numerical approximations, but should be written if needed in terms of traditional constants such as π and e .

b) If $y = Q(x)$, write a formula for x in terms of y .

2. Sketch the region R defined by $0 \leq y \leq 1/x^3$, $1 \leq x \leq 2$.

a) Find (exactly) the number a such that the line $x = a$ divides R into two parts of equal area.

b) Then find (to 3 places) the number b such that the line $y = b$ divides R into two parts of equal area.

3. Let R be the parabolic region in the plane bounded below by the curve $y = x^2$ and above by the line $y = 1$.

a) Sketch R . Set up and evaluate an integral that gives the area of R .

b) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y -axis are squares. Sketch the solid and find its volume.

c) Suppose a solid has base R and the cross-sections of the solid perpendicular to the y -axis are equilateral triangles. Sketch the solid and find its volume.

4. A sort of raindrop is obtained by revolving the profile curve

$$y = \sqrt{x}(x - C)^2 \text{ for } 0 \leq x \leq C$$

about the x -axis. Here C is a positive constant.

a) Sketch the profile curve and the solid of revolution.

b) For which value of C will the raindrop have volume 1? What are the approximate dimensions (length and diameter) of this raindrop?