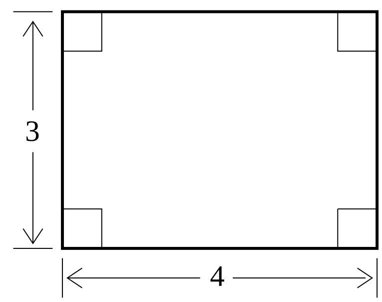
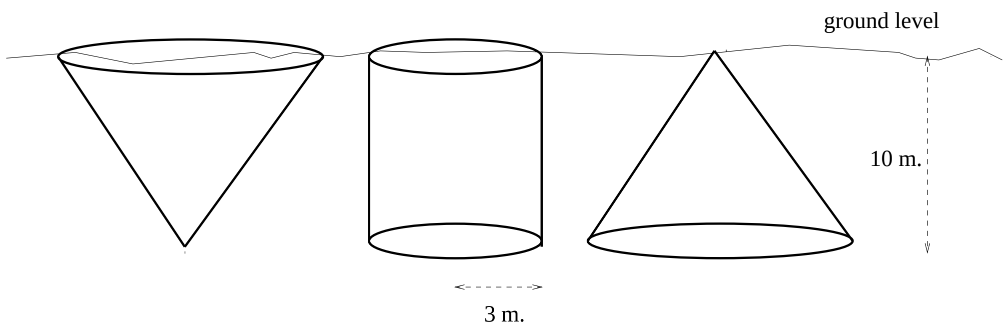
1. A rectangle in the xy-plane has two sides parallel to the y-axis which are 3 units long and two sides parallel to the x-axis which are 4 units long. The lower left-hand corner of the rectangle is at a point (h, k) which is somewhere in the *first quadrant* of the xy-plane. Find one set of values for h and k so that the two solids which are obtained when the rectangle is rotated about the x-axis and the y-axis have the same volume. (There are many answers to this problem.)



2. A homogeneous liquid whose density is 300 kg/m^3 fills three buried containers. Each container, illustrated below, is 10 meters tall. The top of each container is at ground level. All three containers have the *same* volume. The middle container is a cylinder, and the other two are circular cones. Which container needs the *least* amount of work to empty (that is, to pump the liquid to ground level)? Which container needs the *most* work to empty? Justify your assertions by computing the work necessary in each case*.



3. Electrons repel each other with a force which is inversely proportional to the square of the distance between them; call the proportionality constant k in the units to be used. Suppose one electron is fixed at $x = 0$ on the x-axis.

a) Find the work done in moving a second electron along the x-axis from the point $x = 10$ to the point $x = 1$.

b) Find the work done in moving the second electron along the x-axis from the point $x = M$ to the point $x = 1$.

c) What happens to your answer in b) (which should depend on M) as $M \to +\infty$?

4. If a freely falling body starts from rest, then its velocity is given by $v(t) = gt$ and its displacement is given by $s = \frac{1}{2}gt^2$.

a) Suppose the freely falling body starts at rest and falls a distance of $1,000$ feet. Calculate the time T (seconds) that this takes ($g = 32\frac{\text{ft}}{\text{sec}^2}$) and the *time average* of the velocity of the body $v_{\text{time aver}} = \frac{1}{T}\int_0^T v(t)\,dt$. Draw a graph of the function $v(t)$ for $0 \le t \le T$. Find the time t when $v(t) = v_{\text{time aver}}$ and give a graphical interpretation.

This problem is continued on the other side of the page.

* You probably should begin with the cylinder.

b) Find a formula for the velocity as a function $f(s)$ of displacement s, and calculate the *distance average* of the velocity of the body in situation a): $v_{\text{dist aver}} = \frac{1}{1000}\int_0^{1000} f(s)\,ds$. (notice that $v_{\text{dist aver}} \neq v_{\text{time aver}}$). Draw a graph of the function $v = f(s)$ for $0 \leq s \leq 1000$. Find the distance s that the body has fallen when $f(s) = v_{\text{dist aver}}$ and give a graphical interpretation.

Note This is essentially problem 20 of section 6.5 of the text. Everyone who will use statistics (and this means, essentially, every person in this course) should do this problem, because it shows that "even" averages can be difficult to understand.