1. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ both converge (why?). By coincidence it turns out that their sums are both equal to $\ln 2$. (You'll understand this coincidence when we study Taylor series.)

Which series converges "faster" (and so numerically gives a more efficient way to get a numerical approximation for $\ln 2$)? Justify your answer by computing how many terms of each series must be added up to approximate $\ln 2$ with maximum allowed error of 10^{-6}.

2. Consider an infinite series of the form

$$\pm 3 \pm 1 \pm \frac{1}{3} \pm \frac{1}{9} \pm \frac{1}{27} \pm \cdots \pm \frac{1}{3^n} \pm \cdots.$$

The numbers 3, 1, etc., are given but *you* will decide what the signs should be.

a) Can you choose the signs to make the series diverge?

b) Can you choose the signs to make the series sum to 3.5?

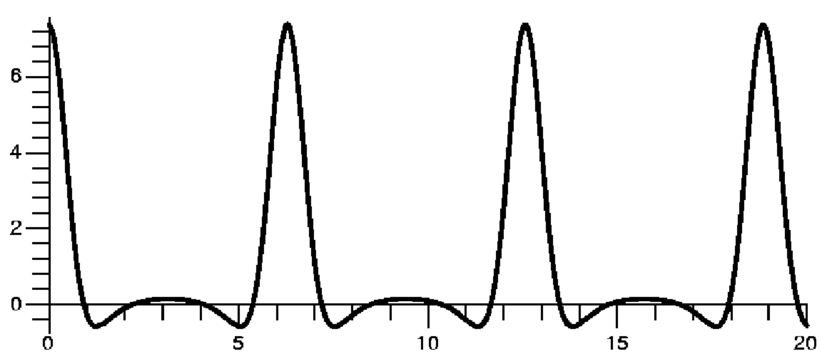
c) Can you choose the signs to make the series sum to 2.25?

In each case, if your answer is "Yes", then specify how to choose the signs; if your answer is "No", then explain.

3. a) Can you find a power series whose interval of convergence is the interval $(0, 1]$, that is, the interval defined by $0 < x \leq 1$? Give an explicit series or explain why you can't.

b) Change the interval in a) to $(0, \infty)$ and answer the same question.

4. Define $f(x)$ with the sum $f(x) = \sum_{n=0}^{\infty} \frac{2^n \cos(nx)}{n!}$. This is not a power series. Below is a graph of the partial sum $s_{100}(x) = \sum_{n=0}^{100} \frac{2^n \cos(nx)}{n!}$ of the series for $0 \leq x \leq 20$.



a) Verify that the series defining $f(x)$ converges for all x.

b) Is the apparent periodicity of the function $f(x)$ actually correct? If yes, explain why.

c) Verify that the actual graph of the function is always within .01 of the graph shown. That is, if x is any real number, then $|f(x) - s_{100}(x)| < .01$.

Possibly useful numbers $2^{100} \approx 1.27 \cdot 10^{30}$ and $2^{101} \approx 2.54 \cdot 10^{30}$. Also, $100! \approx 9.33 \cdot 10^{157}$ and $101! \approx 9.43 \cdot 10^{159}$.