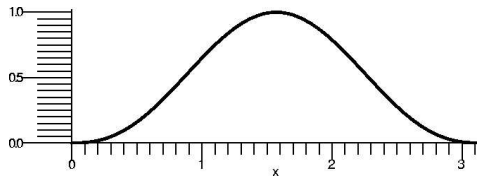(12)   1. Verify that
$$\int_1^3 \frac{2x^2 + 7x + 1}{x(x+1)^2}\,dx = 1 + \ln(2) + \ln(3).$$

(14)   2. A region R in the plane has boundary $y = \left(\sin(x)\right)^{5/2}$ (a graph is shown to the right), the x-axis, $x = 0$, and $x = \pi$.

a) Find the volume of the solid that results from rotating R around the x-axis.

Note The answer will be π multiplied by a rational number. (A rational number is a quotient of integers.)

b) Write a definite integral for the volume of the solid that results from rotating R around the y-axis.

Note You are not asked to compute this volume, just to write a definite integral!

(12)   3. A full bucket of water initially weighs 11 pounds and drips at a constant rate. The bucket itself weighs 1 pound. As the bucket is raised 8 feet, it becomes totally empty. (Between 0 and 8 feet there is always some water in the bucket.) Compute how much work in foot-pounds is done lifting the bucket.
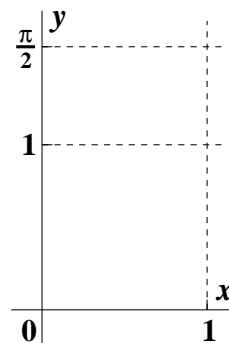
Note The weight of bucket+water is *always* ≥ 1 pound.

(10)   4. Find $\displaystyle\int \frac{x}{\sqrt{1-x}}\,dx$.

(14)   5. The graphs of $y = \arcsin(x)$ and $y = \frac{\pi}{2}x$ intersect twice when x is in the interval $[0,1]$. These graphs form the boundary of a region R.

a) Sketch the region R on the axes to the right.

b) Compute the exact value of the area of R. Be sure to evaluate any trig or inverse trig functions which occur in your answer.

(12)   6. Compute
$$\int_0^\infty (5x + 7)\,e^{-x}dx.$$

Note The answer is an integer. In the computation you will need to apply L'Hôpital's rule. Please be sure to indicate where you are doing this and why L'Hôpital's rule applies.

(14) 7. The parts of this problem are not related.

a) Write the Simpson's Rule estimate for $\int_0^6 \sqrt{x^3 + x}\, dx$ with $n = 6$ subintervals.

Note You are *not* asked to do any arithmetic but asked only to write the sum!

b) The function f here is unknown but the Trapezoid Rule approximation to $\int_1^3 f(x)\, dx$ is 2.25537 with $N = 10$ subintervals. A correct graph of f'' is shown to the right. Use the graph and the approximation to find an interval $[A, B]$ of numbers in which the true value of the definite integral must be found.
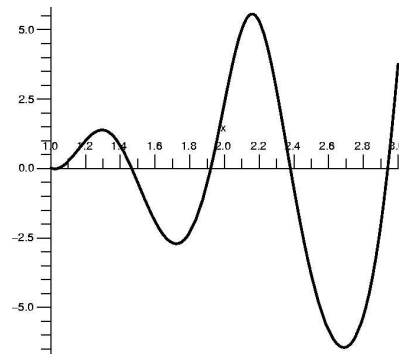
Note This question has many valid answers. Extensive computation is not needed, but your answer must be supported by appropriate evidence.

Graph of $f''(x)$ on $[1, 3]$

(12) 8. Verify that
$$\int_0^1 \frac{1}{\sqrt{16x^2 + 9}}\, dx = \frac{1}{4}\ln(3).$$

First Exam for Math 152

Sections 1, 2, and 3

FEBRUARY 27, 2008

NAME _____

SECTION _____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.

No texts, notes, or calculators other than the
attached formula sheet may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	14	
3	12	
4	10	
5	14	
6	12	
7	14	
8	12	
Total Points Earned:		

Find exact values of standard functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$.
Otherwise do NOT "simplify" your numerical answers!