

Formula Sheet for Math 152, Exam 1

The solutions of $ax^2 + bx + c = 0$ are $x = (-b \pm \sqrt{b^2 - 4ac})/(2a)$.

$$e^{a+b} = e^a e^b, \quad \ln(ab) = (\ln a) + (\ln b), \quad \ln(a^b) = b(\ln a), \quad \ln(1) = 0, \quad \ln(e) = 1$$

$$e^{\ln x} = x, \quad \ln(e^x) = x, \quad \frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(\ln x) = 1/x, \quad \int \frac{du}{u} = \ln |u| + C$$

$$\sin(0) = 0, \quad \sin(\pi/6) = 1/2, \quad \sin(\pi/4) = \sqrt{2}/2, \quad \sin(\pi/3) = \sqrt{3}/2, \quad \sin(\pi/2) = 1$$

$$\cos(0) = 1, \quad \cos(\pi/6) = \sqrt{3}/2, \quad \cos(\pi/4) = \sqrt{2}/2, \quad \cos(\pi/3) = 1/2, \quad \cos(\pi/2) = 0$$

$$\tan x = \sin x / \cos x, \quad \cot x = \cos x / \sin x, \quad \sec x = 1 / \cos x, \quad \csc x = 1 / \sin x$$

$$\cos^2 x + \sin^2 x = 1, \quad 1 + \tan^2 x = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin A \cos B = (1/2)[\sin(A - B) + \sin(A + B)]$$

$$\sin A \sin B = (1/2)[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = (1/2)[\cos(A - B) + \cos(A + B)]$$

$$\sin(2x) = 2 \sin x \cos x, \quad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos^2 x = (1/2)(1 + \cos(2x)), \quad \sin^2 x = (1/2)(1 - \cos(2x))$$

$$\frac{d}{dx}(\sin x) = \cos x, \quad \frac{d}{dx}(\tan x) = \sec^2 x, \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cos x) = -\sin x, \quad \frac{d}{dx}(\cot x) = -\csc^2 x, \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}, \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C, \quad \int \csc x \, dx = \ln |\csc x - \cot x| + C$$

The area between concentric circles is $\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$.

The area of a cylinder is $(2\pi \text{ radius})(\text{height})$.

If the force is constant then work = force \times distance.

The average value of f on $[a, b]$ is $\frac{1}{b-a} \int_a^b f(x) \, dx$.

Midpoint Rule: $\Delta x[f(c_1) + f(c_2) + \dots + f(c_N)]$ where $c_j = (x_{j-1} + x_j)/2$. Trapezoidal

Rule: $\frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_N)]$. Simpson's Rule:

$\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_N)]$.

Sometimes, we write error = approximation - exact value. If error(T_N) and error(M_N) are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then

$$|\text{error}(T_N)| \leq \frac{K_2(b-a)^3}{12N^2} \quad \text{and} \quad |\text{error}(M_N)| \leq \frac{K_2(b-a)^3}{24N^2}, \quad \text{if } |f''(x)| \leq K_2 \text{ for } a \leq x \leq b.$$

If error(S_N) is the error for Simpson's Rule, then

$$|\text{error}(S_N)| \leq \frac{K_4(b-a)^5}{180N^4} \quad \text{where } |f^{(4)}(x)| \leq K_4 \text{ for } a \leq x \leq b.$$

If a rational function is proper and has $(x-a)^M$ in the denominator, then the partial fraction expansion must include $\frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \dots + \frac{A_M}{(x-a)^M}$. If $b > 0$ and a proper

rational function has $(x^2 + b)^N$ in the denominator, then the partial fraction expansion must include $\frac{A_1x + B_1}{x^2 + b} + \frac{A_2x + B_2}{(x^2 + b)^2} + \dots + \frac{A_Nx + B_N}{(x^2 + b)^N}$.