Formula sheet for Math 152, Exam 2

$\int \frac{du}{u} = \ln|u| + C$, $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$, $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$.

$\ln(a^b) = b(\ln a)$, $\tan x = \frac{\sin x}{\cos x}$, $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

---

length $= \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$, surface area $= 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\, dx$

The nth Taylor polynomial of $f(x)$ with center c is $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$. If $|f^{(n+1)}(u)| \le K$ for all u between c and x, then $|f(x) - T_n(x)| \le K\frac{|x-c|^{n+1}}{(n+1)!}$.

$$\lim_{n\to\infty} n^{1/n} = 1 \; ; \quad \lim_{n\to\infty} \frac{x^n}{n!} = 0 \; ; \quad \lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \; ; \quad \lim_{n\to\infty} \frac{n^k}{a^n} = 0 \text{ if } a > 1.$$

$\lim_{n\to\infty} r^n = 0$ when $|r| < 1$; $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ when $|r| < 1$.

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ (and diverges if $p \le 1$).

If the statement $\lim_{n\to\infty} a_n = 0$ is false, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $f(x)$ is a positive decreasing continuous function on $[N, \infty)$ and $a_n = f(n)$ then

$$\int_{n+1}^{\infty} f(x)\, dx \le a_{n+1} + a_{n+2} + a_{n+3} + \cdots \le \int_n^{\infty} f(x)\, dx.$$

In addition, $\sum_{n=N}^{\infty} a_n$ and $\int_N^{\infty} f(x)\, dx$ both converge or both diverge.

If M is a natural number and $0 \le a_n \le b_n$ for $n \ge M$ then: (a) If $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges, (b) if $\sum_{n=1}^{\infty} a_n$ diverges then $\sum_{n=1}^{\infty} b_n$ diverges.

Assume $a_n > 0$, $b_n > 0$, $\lim_{n\to\infty} \frac{a_n}{b_n} = L$. If $0 < L < \infty$ then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge. If $L = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $a_n > 0$, $a_1 \ge a_2 \ge a_3 \ge \cdots$ and $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

$\sum a_n$ converges absolutely when $\sum |a_n|$ converges. $\sum a_n$ converges conditionally when it converges, but does not converge absolutely. If $\sum a_n$ converges absolutely, then $\sum a_n$ converges.

$$\text{If } a_n \neq 0 \text{ and } \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \rho \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } \rho < 1, \\ \sum a_n \text{ diverges if } \rho > 1, \\ \text{the test is inconclusive if } \rho = 1. \end{cases}$$

$$\text{If } \lim_{n\to\infty} |a_n|^{1/n} = L \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

The Taylor series of $f(x)$ with center c is $\displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \; ; \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \; ; \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \; ;$$

$$(1+x)^a = 1 + \sum_{n=1}^{\infty} \left(\frac{a(a-1)(a-2)\cdots(a-n+1)}{n!} \right) x^n \text{ if } |x| < 1.$$