Math 152, Spring 2008, Review Problems for Midterm 2

Your second exam is likely to have problems that do not resemble these review problems. *The official review sheet has been slightly modified for Sections 1, 2, and 3.*

(1) (a) Find the length of the curve $y = x^2/2$, $1 \le x \le 2$. (b) Find the surface area of the surface obtained by rotating $y = (x+7)^3$, $0 \le x \le 1$, about the x-axis.

(2) (a) Find the Maclaurin polynomial $T_{10}(x)$ of $f(x) = x^2\cos(x^3)$ using a Maclaurin polynomial of $\cos(x)$. (b) Find an estimate for $|\sin x - (x - x^3/6)|$ valid over the interval $-1/10 \le x \le 1/10$. Hint for (b): When $f(x) = \sin x$, the polynomial $x - x^3/6$ is a Maclaurin polynomial $T_n(x)$ for *two* values of n. Choose the n which gives you the *smaller* estimate for $|\sin x - (x - x^3/6)|$.

(3) (a) Find the solution of $\dfrac{dy}{dx} = \dfrac{y^3}{1+x^2}$ with initial condition $y(0) = 1$. (b) Find all solutions of $\dfrac{dy}{dx} = xy$.

(4) A room remains at a constant temperature of 20^0 C. A hot drink is brought in at time $t = 0$ hours. At time $t = 2$ hours, the drink's temperature is 60^0 C. At time $t = 6$ hours, the drink's temperature is 50^0 C. Assume that Newton's Law of Cooling is valid, and find the drink's temperature at time $t = 0$ hours.

(5) (a) Find $\lim\limits_{n\to\infty} n\big(\ln(n+10) - \ln n\big)$. (b) Find $\lim\limits_{n\to\infty} \dfrac{n^n}{(n+1)^n}$. Hint for (b): Find $\lim\limits_{n\to\infty} \ln\left(\dfrac{n^n}{(n+1)^n}\right)$ first.

(6) (a) Evaluate $\displaystyle\sum_{n=3}^{\infty} \frac{2^n+1}{(-7)^n}$. (b) Evaluate $\displaystyle\sum_{n=5}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$.

(7) For each series below, determine whether it converges or diverges. Justify your answers, explaining how the various convergence and divergence tests are used.

$$\sum_{n=1}^{\infty}\sqrt{\frac{3n^2-1}{n^4+n^3+2}} \qquad \sum_{n=1}^{\infty}\left(\frac{5n+1}{7n-2}\right)^n \qquad \sum_{n=1}^{\infty}\frac{n^n}{n!5^n} \qquad \sum_{n=1}^{\infty}\frac{1+\cos(n^3+1)}{n^{3/2}}$$

$$\sum_{n=1}^{\infty}\frac{\sin(n^4)}{2^n} \qquad \sum_{n=1}^{\infty}\frac{3^n(n!)^2}{(2n)!} \qquad \sum_{n=2}^{\infty}\frac{1}{n(\ln n)} \qquad \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}} \qquad \sum_{n=1}^{\infty}\frac{n^{10}\cos(n^4+n)}{2^n}$$

(8) (a) Find the interval of convergence of $\displaystyle\sum_{n=1}^{\infty}\frac{5^n(x-3)^n}{n}$. (b) Explain why $(-\infty, 3)$ cannot be the interval of convergence of any power series.

(9) (a) Find the values of x for which the series $\displaystyle\sum_{n=1}^{\infty}\frac{2^n x^n}{n}$ converges. (b) Find N so that $\displaystyle\sum_{n=1}^{N}\frac{1}{n2^n}$ is within .001 of the sum $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n2^n}$.

(10) Find the Maclaurin polynomial $T_8(x)$ of $e^{3x} + e^{-3x}$.