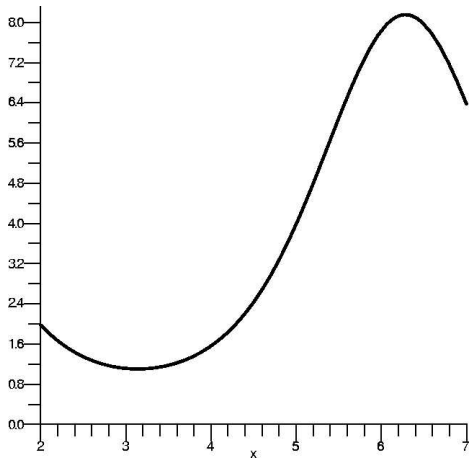
1. Determine how large n has to be in order to approximate the integral $\int_0^1 \cos(x^2)\,dx$, using the Midpoint Rule, with error at most 10^{-6}. Then use this value of n to calculate the integral to this accuracy.

2. Suppose f is defined by $f(x) = 3e^{\cos x}$. `Maple` produced graphs of f and its first four derivatives on the interval $[2, 7]$ (be careful when examining the derivative graphs – look carefully at the vertical scales!). The graph of f is to the right, and the graphs of the first four derivatives of f are on the back of this page. You should assume that the graphs are correct for this problem.



Suppose I is the value of $\displaystyle\int_2^7 f(x)\,dx$.

a) Use the graph of f alone to estimate I.

b) Use the information in the graphs to tell how many subdivisions N are needed so that the Trapezoid Rule approximation T_N will approximate I with error $< 10^{-5}$.

c) Use the information in the graphs to tell how many subdivisions N are needed so that the Simpson's Rule approximation S_N will approximate I with error $< 10^{-5}$.

3. The only information known about a function T and its derivatives is contained in this table:

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4
4	2	5	7

a) Compute $\int_2^3 T'(x)\,dx$.

b) Compute $\int_2^3 T''(x)\,dx$.

c) Compute $\int_2^3 x\,dx$.

d) Compute $\int_2^3 xT''(x)\,dx$. Don't look at b) and c)! Integrate by parts.

e) Compute $\int_2^3 x^2T'''(x)\,dx$. And again and again.

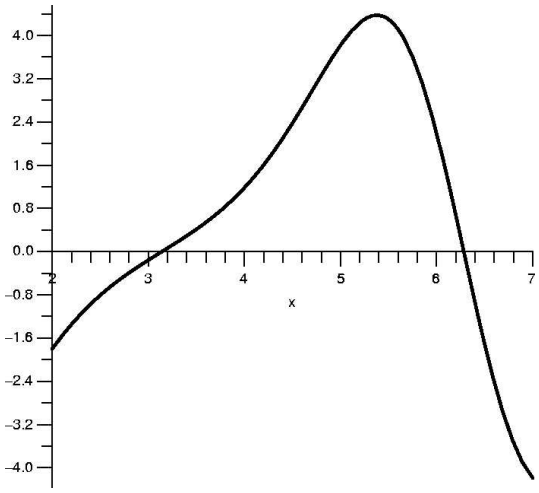
4. Consider the function $f(x) = e^x \sin(Nx)$ on the interval $[0, 1]$ where N is a positive integer.

a) With a sketch or otherwise, describe the graph of this function when $N = 5$, $N = 10$, and $N = 100$.

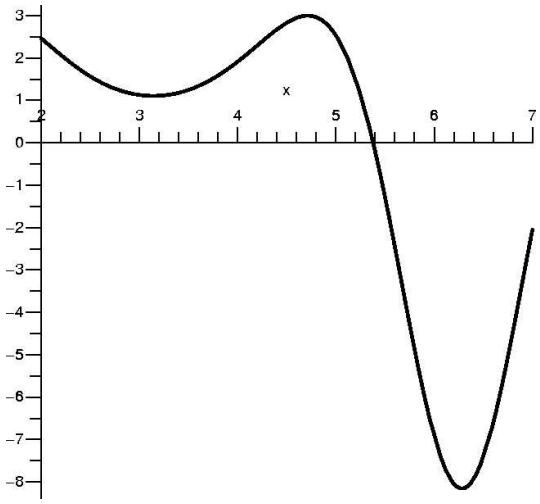
b) Compute $\int_0^1 f(x)\,dx$. Evaluate this integral when $N = 5$, $N = 10$, and $N = 100$.

c) What happens to the graph and to the value of the integral as $N \to \infty$? Does the graph confirm the limiting behavior of the integral's value?

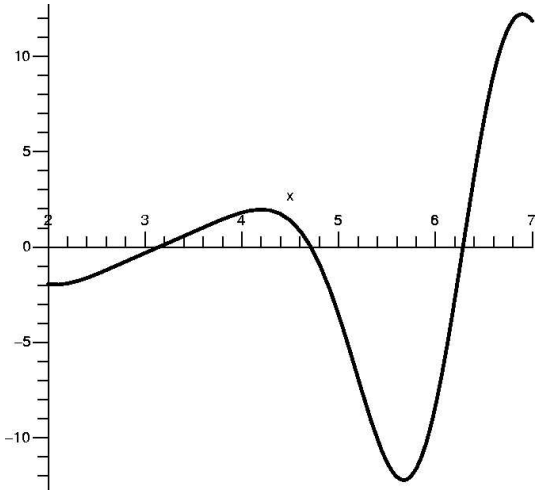
One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage to learn which problem to hand in.



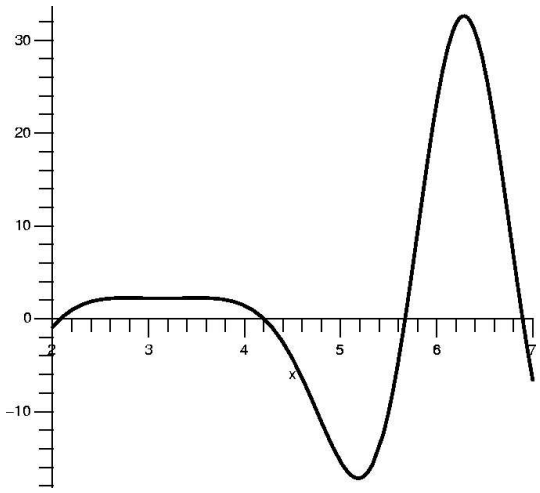
Graph of f'



Graph of f''



Graph of $f^{(3)}$



Graph of $f^{(4)}$