1. Calculate four of the following integrals:

$$\int x\cos x^2\,dx\,;\quad \int x^2\cos x^2\,dx\,;\quad \int x^2\cos x\,dx\,;\quad \int x^2\cos^2 x\,dx\,;\quad \int x\cos^2 x\,dx\,.$$

Comment Most people use *many* parentheses and rewrite the integrands to decrease possible confusion. So $\underline{x^2\cos^2 x}$ becomes $\underline{x^2(\cos x)^2}$ and $\underline{x^2\cos x^2}$ becomes $\underline{x^2\cos(x^2)}$.

2. a) Suppose that m and n are integers. Compute $\int_0^{2\pi}\big(\cos(mx)\big)\big(\cos(nx)\big)\,dx$.
(Be careful: there will be two different results, one when $m=n$ and one when $m\neq n$.)

b) Suppose $f(x)=A\cos(x)+B\cos(2x)+C\cos(3x)$, and that you also know

$$\int_0^{2\pi} f(x)\cos(x)\,dx=5\,;\quad \int_0^{2\pi} f(x)\cos(2x)\,dx=6\,;\quad \int_0^{2\pi} f(x)\cos(3x)\,dx=7\,.$$

Find A and B and C.

Note The ideas of this computation are used often with Fourier series, a standard method of analyzing periodic phenomena.

3. a) Find $\int \frac{e^{2x}}{\sqrt{e^{2x}+1}}dx$.

b) Find $\int \frac{e^{x}}{\sqrt{e^{2x}+1}}dx$.

Comment These antiderivatives may appear similar, but different methods are needed.

4. Compute these two definite integrals exactly:

a) $\int_0^{1/3} 4x\sqrt{1-3x}\,dx$

b) $\int_\pi^{2\pi} x\arcsin\left(\frac{\pi}{x}\right)\,dx$

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage to learn which problem to hand in.