1. Suppose $f(x) = (1-x)^{-1/2} = \frac{1}{\sqrt{1-x}}$.

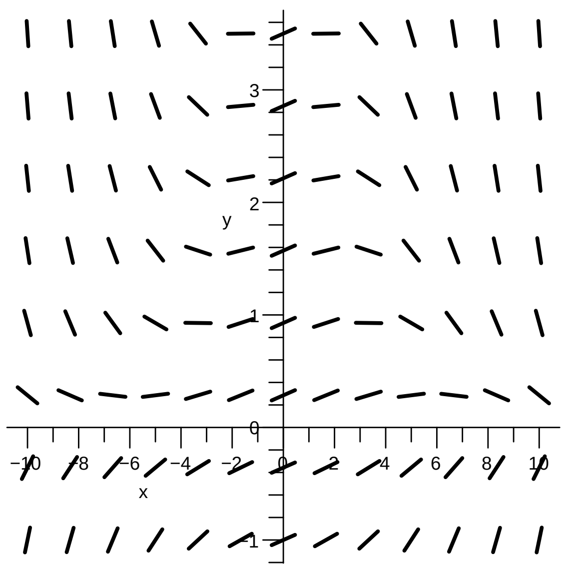
a) Find the fourth Taylor polynomial, $T_4(x)$, centered at $a = 0$ for f.

b) Sketch the graphs of $y = f(x)$ and $y = T_4(x)$ in the window $[-1, 1] \times [0, 3]$.

c) Sketch the graph of $f(x) - T_4(x)$ in the window $[-.5, 5] \times [-.01, .01]$.

d) Use Taylor's inequality (the **Error Bound**) to find an overestimate for $|f(x) - T_4(x)|$ on the interval $[-.5, .5]$. This should be an explicit number valid for all x's in this interval.

2. The horizontal and vertical axes on this graph have different scales. x goes from -10 to 10 and y goes from -1 to 3.5. The graph is a direction field for the differential equation $y' = \frac{1}{10}\left(1 - \frac{1}{10}yx^2\right)$.



a) Sketch the solution curve which passes through $(0, 1)$ **on the graph**.

b) How many critical points does this solution curve seem to have? What types of critical points do they seem to be? If (x_0, y_0) is a critical point, find an exact algebraic relationship between x_0 and y_0.

Comment The equation *can't* be solved in terms of standard functions. Information from the graph and the differential equation should be used.

3. Consider the differential equations

a) $\dfrac{dy}{dx} = 2x + 3y$ b) $\dfrac{dy}{dx} = e^{2x+3y}$ c) $\dfrac{dy}{dx} = x^3y^2$ d) $\dfrac{dy}{dx} = x^2 + y^3$

Two of these are separable. For each of these two separable equations, solve the initial value problem with the initial condition $y(0) = 1$. In each case your solution should be written as $y = f(x)$ where $f(x)$ is a formula. Choose one of the *non-separable* equations and explain carefully why it is *not* separable.

4. A 200-gal tank contains 100 gal of water with a salt concentration of 0.1lb/gal. Water with a salt concentration of 0.4 lb/gal flows in the tank at a rate of 20 gal/min. The fluid is mixed instantaneously, and water is pumped out at a rate of 10 gal/min. Let $y(t)$ be the amount of salt in the tank at time t.

a) Set up and solve the differential equation for $y(t)$.

b) What is the salt concentration when the tank overflows?

This is a problem from the textbook: #28 in section 9.5.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage to learn which problem to hand in.