1. Suppose A and B are positive numbers. Decide whether the following sequences converge. If they converge, try to find their limits. Your answers may involve both numbers and their relationship.

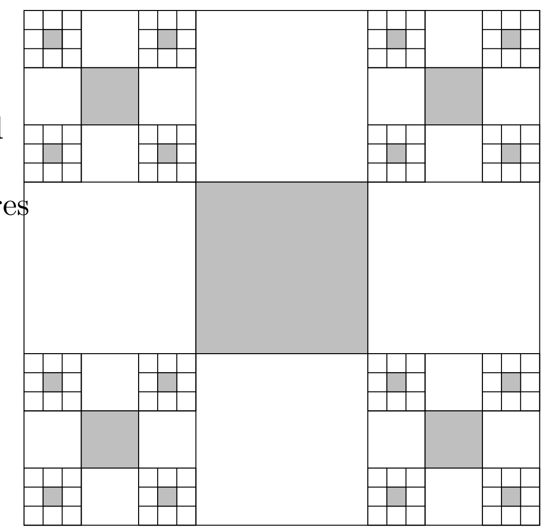
a) $c_n = \sqrt[n]{A^n + B^n}$ b) $d_n = \sqrt[n]{A^n + B}$

Hint Experiment! Choose various values of A and B and compute the first five or ten terms of each sequence. Then verify your guesses in general with algebra and calculus.

2. a) Enter the number 5 in a calculator showing 10 decimal digits after the decimal point. Press the square root button 20 times. The result will be **1.00000 15348** . Subtract 1 and multiply by 1,048,576 to get **1.60943 91475** *but* the same calculator will declare that $\ln 5$ is **1.60943 79124**. Since 1,048,576 is 2^{20}, this is *not* a concidence. Explain.

b) Given a positive number, x, outline a strategy for computing $\ln x$ only with the arithmetic operations ($+$, $\times$, $-$, $/$) and square root ($\sqrt{\ }$). Your strategy should involve asserting (and verifying) that a certain sequence which can be easily computed with the listed operations always converges to $\ln x$.

3. A 1×1 square is "dissected" by three equally spaced horizontal lines and by three equally spaced vertical lines. The central square is shaded. Then the bordering Northeast, Northwest, Southeast, and Southwest squares are similarly dissected, with the central square shaded. Each of *those* dissected squares has a similar process done to their borders, etc. The diagram to the right shows this only for the first three steps but it is supposed to continue indefinitely.



a) How many new shaded squares are introduced at the n^{th} step? (There is one shaded square at the first step.) What is the side length of the squares which are introduced at the n^{th} step?

b) What is the sum, as n goes from 1 to ∞, of the shaded area (all the shaded squares)? What is the sum, as n goes from 1 to ∞, of the perimeters of all the shaded squares?

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage to learn which problem to hand in.