(14) 1. Use the method of partial fractions to verify that

$$\int_0^1 \frac{1}{(x+1)(x^2+1)}\,dx = \frac{1}{4}\ln 2 + \frac{1}{8}\pi$$

(16) 2. a) The base of a solid is the region bounded by the parabola $y = x^2$ and the line $y = 4$. Cross-sections of the solid by planes perpendicular to the $x-$axis are squares. Find the volume of the solid.

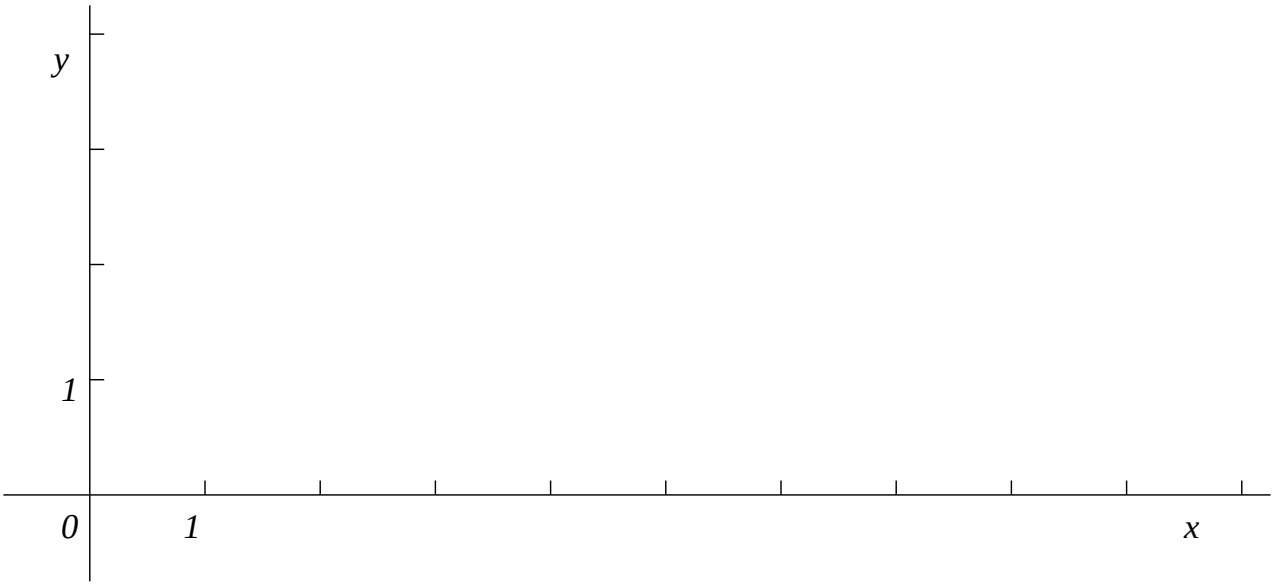
b) The base of a solid is the region bounded by the parabola $y = x^2$ and the line $y = 4$. Cross-sections of the solid by planes perpendicular to the $y-$axis are squares. Find the volume of the solid.

(12) 3. Use integration by parts followed by a substitution (or a substitution followed by integration by parts) to verify that

$$\int_0^1 e^{\sqrt{x}}\,dx = 2$$

(16) 4. Suppose $\mathcal{R}$ is the region in the first quadrant bounded by the two curves $y = 4e^{-2x}$ and $y = 4e^{-3x}$.

a) Sketch that part of the region between $x = 0$ and $x = 10$ on the axes given.

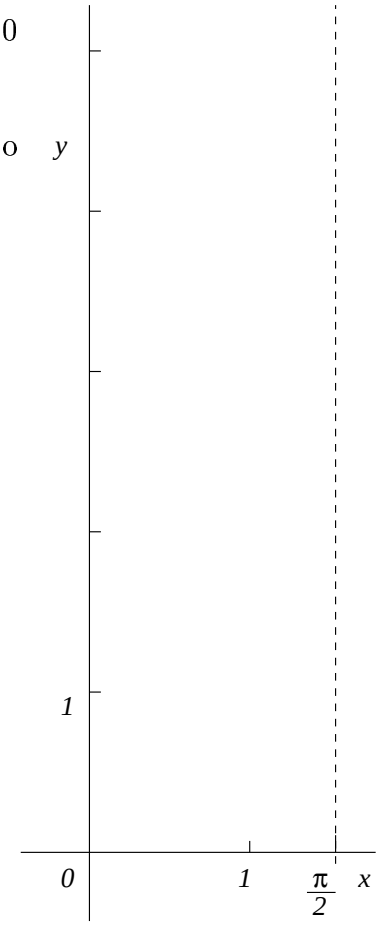


b) Compute the area of the whole region $\mathcal{R}$ (out to ∞) if it is finite.

(16) 5. Suppose $\mathcal{S}$ is the three-sided region in the first quadrant bounded by the $y-$axis and the two curves $y = \tan x$ and $y = \sec x$.

a) Sketch that part of the region between $y = 0$ and $y = 5$ on the axes given.

b) Compute the area of the whole region $\mathcal{S}$ (up to ∞) if it is finite.



(12) 6. a) Suppose m and n are positive integers. Find a reduction formula for

$$\int x^m (\ln x)^n \, dx$$

(Here the object is to reduce n, since if we can push n to 0 we'll just have a polynomial to integrate, which is easy.)

b) Use the formula obtained in a) to compute

$$\int x^{20} (\ln x)^2 \, dx$$

(14) 7. This problem analyzes the computation needed to estimate the definite integral

$$\int_0^1 \frac{1}{9} \sin\left(x^3\right) dx$$

a) Find n (the number of subdivisions) so that the Trapezoidal Rule estimate will be within 10^{-6} of the true value of the definite integral. (You may use the error bound $\dfrac{K(b-a)^3}{12n^2}$ where K is an overestimate of the magnitude of the second derivative.)

DO NOT COMPUTE THE TRAPEZOIDAL RULE ESTIMATE.

b) Find n (the number of subdivisions) so that the Simpson's Rule estimate will be within 10^{-6} of the true value of the definite integral. (You may use the error bound $\frac{J(b-a)^5}{180n^4}$ where J is an overestimate of the magnitude of the fourth derivative.)

DO NOT COMPUTE THE SIMPSON'S RULE ESTIMATE.

EXAM 1 for MATH 192:03

October 10, 1996

NAME (*please print*): _____

SIGNATURE: _____

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use <u>one</u> sheet of notes and any standard calculator without a QWERTY keypad on this exam. You may use <u>no</u> other materials.

Problem Number	Possible Points	Points Earned:
1	14	
2	16	
3	12	
4	16	
5	16	
6	12	
7	14	
Total Points Earned:		