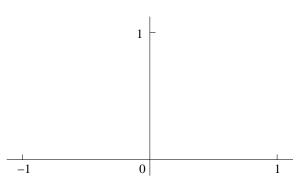
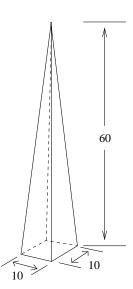
- (16) 1. a) Compute the area enclosed by  $y = 1 x^2$  and the x-axis.
  - b) Suppose a is an unspecified positive number. Sketch and label the curves  $y = 1 x^2$  and  $y = ax^2$  on the axes given. Find the coordinates of the points of intersection of  $y = 1 x^2$  and  $y = ax^2$ , and label these points on your sketch.
  - c) Find a so that the area enclosed by  $y = 1-x^2$  and  $y = ax^2$  is half of the area computed in part a).



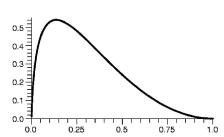
- (12) 2. A flat-sided monolith\* is 60 feet tall with a square base that is 10 feet on each side. What is the volume of the monolith?
- (18) 3. a) Here's a formula from the Tables of Indefinite Integrals by G. Petit Bois (1906):  $\int \frac{x^2}{x^3 + 5x^2 + 8x + 4} dx = \log(x+1) + \frac{4}{x+2}.$  Please verify this formula using the method of partial fractions.
  - b) Here's another formula from the same text:  $\int \frac{x^2}{x^3+x^2+x+1} dx = \frac{1}{2} \log \left( (x+1)\sqrt{x^2+1} \right) \frac{1}{2} \arctan x$ . Again, please verify this formula using the method of partial fractions.



- (18) 4. a) Compute  $\int_0^1 x \arcsin(x^2) dx$ .
  - b) Compute  $\int_0^1 \frac{e^{2x}}{e^x+1} dx$ .
- (16) 5. In this problem,  $f(x) = x (\ln x)^2$ .
  - a) Verify that  $\lim_{x\to 0^+} f(x) = 0$ .

**Hint** Write the limit so you can apply L'H, but be sure to indicate why you need L'H whenever you use it.

- b) Carefully compute the improper integral  $\int_0^1 f(x) dx$ . Indicate why the limits you need exist, and what these limits are.
- c) Here's a graph of  $x(\ln x)^2$  drawn by Maple on the interval [0,1]. Does this graph (approximately) confirm your computation in b)? Why?



## \* monolith

- 1. A large block of stone, especially one used in architecture or sculpture.
- 2. Something, such as a column or monument, made from one large block of stone.

(10) 6. The integral  $\int_2^\infty \frac{1}{4x^{1/3} + 5x^{1/4}} dx$  diverges to  $\infty$ . Find some A > 0 so that  $\int_2^A \frac{1}{4x^{1/3} + 5x^{1/4}} dx > 10^{10}$ .

**Comment** You are *not* asked to find an explicit antiderivative of  $\frac{1}{4x^{1/3} + 5x^{1/4}}$ \*\*. You are *not* asked to find a "best possible" A.

You are asked to find a valid A and to support your answer with some reasoning.

(10) 7. This problem analyzes the computation needed to estimate the definite integral  $\int_0^2 x \left(1+x^3\right)^{3/2} dx$  using the Trapezoidal Rule. Find n (the number of subdivisions) so that the Trapezoidal Rule estimate will be within  $10^{-6}$  of the true value of the definite integral. (You may use the error bound  $\frac{K(b-a)^3}{12n^2}$  where K is an overestimate of the magnitude of the second derivative.)

**Comment** You are *not* asked to compute this approximation to the definite integral. You are *not* asked to find a "best possible" n.

You are asked to find a valid n and to support your answer with some reasoning.

<sup>\*\*</sup> Maple showed an antiderivative with 23 complicated terms. This was not useful.

## First Exam for Math 192, section 1

October 17, 2005

NAME		

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes other than the distributed formula sheet may be used on this exam.

No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	16	
2	12	
3	18	
4	18	
5	16	
6	10	
7	10	
Total Poi	nts Earned:	