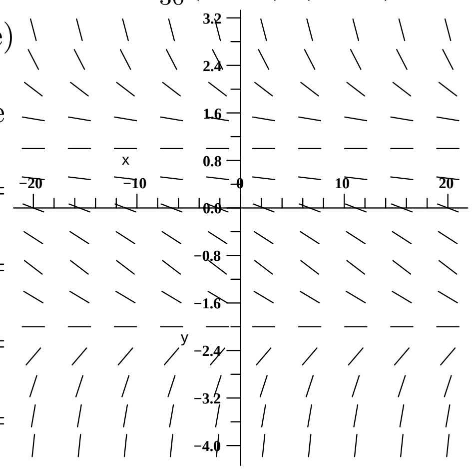
(12) 1. The horizontal and vertical axes on the graph below have different scales. The graph is a direction field for to the autonomous differential equation $y' = -\frac{1}{30}(y-1)^2(2+y)$.



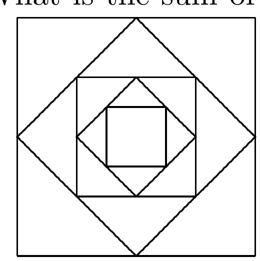
a) Find the equilibrium solutions (where y doesn't change) for this differential equation.
b) Sketch solution curves on the axis above through these points. Find the indicated limits:

- $(0,0)$. Label this curve **A**. On curve **A**, $\lim_{x\to\infty} y(x) =$ __________.
- $(0,2)$. Label this curve **B**. On curve **B**, $\lim_{x\to\infty} y(x) =$ __________.
- $(0,-3)$. Label this curve **C**. On curve **C**, $\lim_{x\to\infty} y(x) =$ __________.
- $(1,0)$. Label this curve **D**. On curve **D**, $\lim_{x\to\infty} y(x) =$ __________.

c) One of the equilibrium solutions is a *stable* equilibrium. Which one?

(15) 2. Consider the differential equation $y' = \frac{1+y}{x^3}$.
a) Find the general solution.
b) Find a particular solution with $y(1) = 0$ and describe its domain.
Comment The domain is slightly tricky.

(15) 3. The parts of this problem are not related.
a) The first term of a geometric series is 5 and the fourth term is 3. What is the sum of the geometric series?
b) An infinite sequence of squares is drawn (the first five are shown), with the midpoints of the sides of one being the vertices of the next. The outermost square has sides which are 1 unit long. What is the <u>sum</u> of the <u>perimeters</u> of all of the squares?



(12) 4. a) What is the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{3^n}{n^2} x^n$?
b) What is the behavior of the power series (divergence, absolute or conditional convergence) on the boundary points of the interval of convergence?

(12) 5. In this problem you will consider the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^2}$.
a) Briefly explain why this series <u>converges</u>.
b) `Maple` gives the approximate value .84776 to 5 digit accuracy for the sum of this series. Find a specific partial sum which is guaranteed to give this number to 5 digit accuracy. Give evidence supporting your assertion.
Comment You are *not* asked for the best possible partial sum satisfying the indicated requirement! You must, however, give supporting evidence for the partial sum you give.

(12) 6. In this problem you will consider the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$.

a) Briefly explain why this series <u>diverges</u>.

b) According to `Maple`, the 10,000$^{\text{th}}$ partial sum of this series is about 4.74561. Are the partial sums of this series unbounded? If yes, find a specific partial sum which is guaranteed to be greater than 100. Give evidence supporting your assertion.

Comment You are *not* asked for the best possible partial sum satisfying the indicated requirement! You must, however, give supporting evidence for the partial sum you give.

(12) 7. In this problem you will consider the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2^n}$.

a) Briefly explain why this series <u>converges</u>.

b) `Maple` gives the approximate value .73601 to 5 digit accuracy for the sum of this series. Find a specific partial sum which is guaranteed to give this number to 5 digit accuracy. Give evidence supporting your assertion.

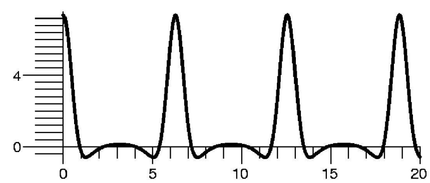
Comment You are *not* asked for the best possible partial sum satisfying the indicated requirement! You must, however, give supporting evidence for the partial sum you give.

(12) 8. This problem investigates the function $f(x)$ defined by the sum $f(x) = \sum_{n=0}^{\infty} \frac{2^n \cos(nx)}{n!}$. This is <u>not</u> a power series. Below is a graph of a high partial sum of the series for $0 \le x \le 20$.

a) Does this series converge for all x? If yes, explain why.

b) Is the apparent periodicity of the function actually correct? If yes, explain why.

c) Is the function actually bounded? That is, can you find some positive number B so that $|f(x)| \le B$ for all x?



Comment You are *not* asked for the best possible bound. If you think a bound exists, find one such bound, and give supporting evidence for your assertion. Otherwise, explain why a bound does not exist.

(12) 9. True or false? If false, give an example to show that the implication is not true. If true, briefly explain why.

a) Suppose that all of the a_n's are positive. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

b) Suppose that all of the a_n's are positive. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges.

Second Exam for Math 192, section 1

November 22, 2005

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	15	
3	15	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
Total Points Earned:		