(10) 1. Compute $\displaystyle\int \frac{dx}{(x+3)x}$.

(10) 2. Compute $\displaystyle\int_1^2 x^2 \ln x \, dx$.

(6) 3. Describe both the domain and the range of $\ln(1-x^2)$ as precisely as possible.

(14) 4. Describe how to approximate $e^{\frac{1}{10}}$ with an error of size at most .0001 using a partial sum of a Taylor series. You must be sure to give an explicit error estimate, and to write an explicit partial sum which correctly approximates $e^{\frac{1}{10}}$. You may use the fact that $e < 3$. (Do the arithmetic needed for the error estimate; do not do the arithmetic involved in the partial sum!)

(10) 5. A curve is defined in parametric form by the equations $x = e^{2t}\cos t$, $y = \sin(3t) - 2$.

(a) Find the equation of the line tangent to this curve at the point given by $t = 0$.

(b) Set up, but do not simplify or evaluate, an integral giving the length of this curve as t varies from 0 to π.

(10) 6. Compute $\displaystyle\int \frac{1}{\sqrt{3+2x-x^2}}\, dx$.

(10) 7. (a) Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converge?

(b) Does $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converge?

(10) 8. Let f be a function whose fourth derivative satisfies the inequality $|f^{(4)}(x)| \le \dfrac{1}{3+5x^2}$ for all x and let $p_3(x)$ be the third degree Taylor polynomial for f centered at 0. Find an estimate for the error obtained when using $p_3(2)$ in place of $f(2)$.

(16) 9. Let $f(x) = x^2 e^{-2x}$. Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. For which values of x does $f(x)$ have a local maximum? For which values of x does $f(x)$ have a local minimum? Sketch the graph of $y = f(x)$ using all of the information you have found (you do not need to worry about concavity).

(10) 10. (a) Is $\displaystyle\int_1^{\infty} \frac{4+\sin x}{x^2}\, dx$ finite?

(b) Is $\displaystyle\int_1^{\infty} \frac{4+\sin x}{x^2}\, dx < 6$?

(c) Is $\displaystyle\int_1^{\infty} \frac{4+\sin x}{x^2}\, dx > 2$?

(10) 11. Use the Taylor series for cosine to write $\displaystyle\int_0^{\frac{1}{2}} \cos(x^3)\, dx$ as the sum of an infinite series.

(10) 12. Determine all values of x for which $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^2 x^{2n}}{2^n}$ converges.

(10) 13. Suppose $f(x) = (8+x)^{1/3}$. What is $p_3(x)$, the third degree Taylor polynomial for f centered at 0?

(12) 14. Sketch the polar curve $r = 3(1+\sin\theta)$ and find the area inside it.

(12) 15. Find a third degree polynomial approximation to $\tan x$ for x near 0. Use this approximation to decide if the integral $\displaystyle\int_0^{\frac{1}{10}} \frac{\tan x}{x}\,dx$ converges.

(10) 16. (a) What is $\lim_{n\to\infty} ne^{-n}$?

(b) What is $\lim_{n\to\infty} \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$?

(12) 17. Suppose $K_n = \displaystyle\int_0^{\pi} x\sin(nx)\,dx$. Compute K_n. What is $\lim_{n\to\infty} K_n$?

(8) 18. Compute $\displaystyle\int_0^1 \frac{x^2}{1+x^2}\,dx$.

(10) 19. The pressure P and volume V of a certain gas are related by the differential equation $P + 3V\dfrac{dP}{dV} = 0$. Solve for P as a function of V.

FINAL EXAM for MATH 192

December 20, 1993

NAME (*please print*): _____

SIGNATURE: _____

Problem Number	Possible Points	Points Earned:
1	10	
2	10	
3	6	
4	14	
5	10	
6	10	
7	10	
8	10	
9	16	
10	10	
11	10	
12	10	
13	10	
14	12	
15	12	
16	10	
17	12	
18	8	
19	10	
Total Points Earned:		

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

NO $\left\{ \begin{array}{c} \textbf{books} \\ \textbf{notes} \\ \textbf{calculators} \end{array} \right\}$ **of any kind may be used.**

All answers **<u>should</u>** be left in "unsimplified" form – that is, $15^2 + (.07) \cdot (93.7)$ is preferred to 231.559. You are expected to know, however, simple values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$.