Formula sheet for Math 152, Exam 1

$$\cos^2 x + \sin^2 x = 1 \ , \quad 1 + \tan^2 x = \sec^2 x \ , \quad 1 + \cot^2 x = \csc^2 x$$
$$\cos(2x) = \cos^2 x - \sin^2 x \ , \quad \sin(2x) = 2 \sin x \cos x$$
$$\cos^2 x = \frac{1 + \cos(2x)}{2} \ , \quad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \ , \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x \ , \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}} \ , \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$
$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \ , \quad \int \csc x \, dx = \ln|\csc x - \cot x| + C$$

The area between concentric circles is $\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$. The area of a cylinder is $(2\pi \,\text{radius})(\text{height})$.

If the force is constant then work = force $\times$ distance.

The average value of f on $[a, b]$ is $\dfrac{1}{b - a} \displaystyle\int_a^b f(x)\, dx$.

If a proper rational function has $(ax + b)^r$ in the denominator, then its partial fraction expansion must include the sum $\dfrac{A_1}{ax + b} + \dfrac{A_2}{(ax + b)^2} + \cdots + \dfrac{A_r}{(ax + b)^r}$.

If $ax^2 + bx + c$ is irreducible and a proper rational function has $(ax^2 + bx + c)^r$ in the denominator, then its partial fraction expansion must include the sum

$$\frac{A_1 x + B_1}{ax^2 + bx + c} + \frac{A_2 x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_r x + B_r}{(ax^2 + bx + c)^r}.$$

The Midpoint Rule is $\Delta x[f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_n)]$ where $\overline{x}_i$ is the midpoint of $[x_{i-1}, x_i]$.

The Trapezoidal Rule is $\dfrac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)]$.

Simpson's Rule is

$$\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)].$$

If E_T and E_M are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then

$$|E_T| \le \frac{K(b - a)^3}{12n^2} \quad \text{and} \quad |E_M| \le \frac{K(b - a)^3}{24n^2} \quad \text{where} \quad |f''(x)| \le K \quad \text{for} \quad a \le x \le b.$$

If E_S is the error for Simpson's Rule then $|E_S| \le \dfrac{K(b - a)^5}{180n^4}$ where $|f^{(4)}(x)| \le K$ for $a \le x \le b$.

$$\text{length} = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \ , \quad \text{surface area} = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2}\, dx$$