Formula sheets for the Final Exam in Math 152

$$\cos^2 x + \sin^2 x = 1 \ , \quad 1 + \tan^2 x = \sec^2 x \ , \quad 1 + \cot^2 x = \csc^2 x$$

$$\cos(2x) = \cos^2 x - \sin^2 x \ , \quad \sin(2x) = 2\sin x \cos x$$

$$\cos^2 x = \frac{1+\cos(2x)}{2} \ , \quad \sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \ , \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \ , \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \ , \quad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C \ , \quad \int \csc x \, dx = \ln|\csc x - \cot x| + C$$

The area between concentric circles is $\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2$. The area of a cylinder is $(2\pi \, \text{radius})(\text{height})$.

If the force is constant then work = force $\times$ distance.

The average value of f on $[a,b]$ is $\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

If a proper rational function has $(ax+b)^r$ in the denominator, then its partial fraction expansion must include the sum $\dfrac{A_1}{ax+b} + \dfrac{A_2}{(ax+b)^2} + \cdots + \dfrac{A_r}{(ax+b)^r}$.

If ax^2+bx+c is irreducible and a proper rational function has $(ax^2+bx+c)^r$ in the denominator, then its partial fraction expansion must include the sum

$$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \cdots + \frac{A_rx+B_r}{(ax^2+bx+c)^r}.$$

The Midpoint Rule is $\Delta x[f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)]$ where $\overline{x}_i$ is the midpoint of $[x_{i-1},x_i]$.

The Trapezoidal Rule is $\dfrac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)]$.

Simpson's Rule is

$$\frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)].$$

If E_T and E_M are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then

$$|E_T| \le \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \le \frac{K(b-a)^3}{24n^2} \quad \text{where} \quad |f''(x)| \le K \quad \text{for} \quad a \le x \le b.$$

If E_S is the error for Simpson's Rule then $|E_S| \le \dfrac{K(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \le K$ for $a \le x \le b$.

$$\text{length} = \int_a^b \sqrt{1+[f'(x)]^2}\,dx \ , \ \text{surface area} = \int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx.$$

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$$\lim_{n\to\infty} n^{1/n} = 1 \;; \quad \lim_{n\to\infty} \frac{x^n}{n!} = 0 \;; \quad \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \;; \quad \lim_{n\to\infty} \frac{n^k}{a^n} = 0 \text{ if } a > 1.$$

$$\lim_{n\to\infty} c^n = 0 \text{ if } |c| < 1 \;; \quad \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \text{ if } |c| < 1.$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges if } p > 1 \text{ (and diverges if } p \le 1).$$

If the statement $\lim\limits_{n\to\infty} a_n = 0$ is false, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

If $0 < a_n$, $0 < b_n$, $0 < \lim\limits_{n\to\infty} \dfrac{a_n}{b_n} < \infty$, then $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ both converge or both diverge.

If $f(x)$ is a positive decreasing continuous function and $a_n = f(n)$ then
$\int_{n+1}^{\infty} f(x)\,dx \le a_{n+1} + a_{n+2} + a_{n+3} + \cdots \le \int_n^{\infty} f(x)\,dx$.

If $a_n > 0$, $a_1 \ge a_2 \ge a_3 \ge \cdots$ and $\lim\limits_{n\to\infty} a_n = 0$ then $\sum\limits_{n=1}^{\infty} (-1)^n a_n$ converges.

$\sum a_n$ converges absolutely when $\sum |a_n|$ converges. $\sum a_n$ converges conditionally when it converges, but does not converge absolutely. If $\sum a_n$ converges absolutely, then $\sum a_n$ converges.

If $a_n > 0$ and $\lim\limits_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = L$ then
$$\begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

If $a_n \ge 0$ and $\lim\limits_{n\to\infty} |a_n|^{1/n} = L$ then
$$\begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

$T_n(x) = \sum\limits_{k=0}^{n} \dfrac{f^{(k)}(a)}{k!}(x-a)^k$, $R_n(x) = f(x) - T_n(x)$, $\sum\limits_{n=0}^{\infty} \dfrac{f^{(n)}(a)}{n!}(x-a)^n$ is the Taylor series of $f(x)$ with center a.

If $|f^{(n+1)}(x)| \le M$ for $|x-a| \le d$, then $|R_n(x)| \le \dfrac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \le d$.

$$e^{ix} = \cos x + i \sin x \;; \quad (1+x)^k = 1 + \sum_{n=1}^{\infty} \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} x^n \text{ if } |x| < 1.$$

$$\text{length} = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt, \quad \text{length} = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta, \quad \text{area} = \int_a^b \frac{r^2}{2}\, d\theta.$$