640:192:01 Part III: playing with calculus on `maple` 9/1/2005

The basic calculus commands do differentiation and integration. Let's try them.

diff(3xˆ7 – 22.1xˆ2,x) **RET**

int(x∗sqrt(x+2),x) **RET**

This integration is not easy: `maple` either substituted or it integrated by parts (it can do both). Writing the ∗ in forces `maple` to recognize variables. I've done some "experiments" and the results suggest that writing ∗'s rather than having `maple` guess is most useful. `maple` likely knows all the functions you do, and many others. The function $\sin\left(x^2\sqrt{x+1}\right)$ and the calculus rules are known. Let's assign this expression a name and then play.

Q:=sin(xˆ2 ∗ sqrt(x+1)) **RET**

diff(Q,x) **RET**

This should get the first derivative. How many ways can you think of to get the second derivative? First, immediately after the command and response above, type

diff(% ,x) **RET**

This will do it. For an independent computation, try the command line

diff(Q,x,x) **RET**

How about the tenth derivative? First type

x$10 **RET**

to see what *$* means. Now you find the tenth derivative of the function $\sin\left(x^2\sqrt{x+1}\right)$. Realize (even if the tenth derivative of this function is needed) why people end commands with *:* (which turns off the output) rather than displaying the output. You may want the results of a computation, but you may not have the need or desire to actually look at it!

What is the coefficient of x^3 in the seventh derivative of $\left(x^2+\frac{1}{x^2}\right)^5$? First compute the indicated derivative. You'll get a mess. Then have `maple` massage the result algebraically so you can read off the answer. I'm an amateur and first tried *expand(%)* and I also tried *simplify(%)* and the results were different. Another way is to first *expand* $\left(x^2+\frac{1}{x^2}\right)^5$ and then differentiate the result seven times. The answer should be the same!

Let's look at integration a bit more closely. Define V to be $e^{\sin x}$:

V:=exp(sin(x)) **RET**

Now let's integrate it. First (try this carefully!) type

int(V,w) **RET**

and explain the result to yourself. Remember, a program will do what you tell it to do! Now try

int(V,x) **RET**

and you may wait a bit and then have something else to explain. `maple` has the usual integration algorithms and many, many other antidifferentiation tricks. An answer like this is a fairly good hint that it "can't be done": that is, the antiderivative can't be expressed in terms of familiar functions with familiar ways of combining them, including sum, product, composition, ... even using the rather large collection of functions known to `maple`. Such results can be proved!

We can also compute definite integrals. For example,

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int(x^3,x=1/7..b) **RET**

computes $\int_{\frac{1}{7}}^{b} x^3 dx$ (if you want it!). `maple` indicates a range (for integration and for other purposes) by the notation *variable=lower_limit..upper_limit*.

Remember V? Be sure `maple` does (check by typing V) and compute $\int_0^1 e^{\sin x}\, dx$ by typing

int(V,x=0..1) **RET**

and consider the result. Disappointment is decreased if we follow that answer with

evalf(%); **RET**

You can evaluate V itself with a command like

subs({x=3},V) **RET**

followed by

evalf(%) **RET**

If you're not scared, you could have done this all together by typing

evalf(subs({x=3},V)) **RET**

but sometimes I get confused by the matching required (in count and type) of all the parentheses. We could similarly evaluate a derivative of V by differentiating with *diff*, *subs*tituting, and then *evalf*uating. Or we could define our own functions. Initially the syntax may seem burdensome, but here is a simple example.

N:=x–>arctan(x^3) **RET**

So: call N the function which takes the input x and assigns to it $\arctan(x^3)$. Try this

*N(2);N(5*z)* **RET**

which shows that `maple` believes N is a function. Now type

diff(N(x),x); **RET**

That's an **expression** and not a function. There's no way to "plug in" $K + 3$ easily in it (yes, we could use the *subs* command, but that's cumbersome). So `maple` has another way to differentiate functions (such as N) rather than expressions (such as V). Try

D(N); **RET**

and view the result. Indeed: call it by a new name, say, M:

M:=D(N); **RET**

Now evaluate $M(3)$ and $M(K + 3)$.

Please check the difference between

int(N(x),x); **RET**

and

int(N,x); **RET**

One of these has an answer that's fun to me (because I didn't have to compute it and can appreciate the work involved!) and the other seems silly: the function N hasn't been told what to evaluate, so it can't be antidifferentiated.

The difference between "expressions" like V and "functions" like N can be subtle. Expressions seem more static, while functions have a more dynamic aspect – substituting is built into their structure.

`maple` also knows about such calculus topics as limits, sums, and series. If you need to work with any of these, at that time please try *help(limit)* and *help(sum)* and *help(series)*. I almost always look at the examples first. Don't be afraid to try experiments, and look at the *help* pages if the command you are investigating doesn't do exactly what you want. It's time to graph.