Please write solutions to two of these problems. Solutions are due on Monday, September 19. Solutions will be graded both for mathematical content and for exposition, which should be in complete English sentences. You need not show every computational detail, but please give enough information to allow readers to follow your reasoning. You may also include appropriate diagrams, which should be carefully drawn and labeled.

For this problem set, students *must* work in teams and each team will hand in one set of solutions. Each team should ideally have two students (let me know if this causes difficulty). I also would like students whose family language is *not* English to work with students whose family language is English. The grades for the members of each team will be the same. Team members should decide how to divide their effort. Certainly you may discuss these problems with each other and with me and Mr. Scheinberg.

1. a) Consider the function $V(x) = x^{1/x}$, whose domain is $(0, \infty)$. Compute $V'(x)$ and find any critical points of V. Either compute V'' or study V' more closely to conclude information about the nature of the critical point(s?) of V. Also investigate the limits of $V(x)$ as $x \to \infty$ and as $x \to 0^+$.

c) Compute $(10{,}000{,}000{,}000)^{\left(\frac{1}{10{,}000{,}000{,}000}\right)}$ and $\ln(10)$. (You'd better have *many* digits of accuracy in the first number.) Use calculus to explain any interesting coincidences in your answers.

2. A sort of raindrop is obtained by revolving the profile curve $y = \sqrt{x}(x-C)^2$ for $0 \le x \le C$ about the x-axis. Here C is a positive constant.

a) Sketch the profile curve and the solid of revolution.

b) For which value of C will the raindrop have volume 1? What are the rough dimensions (length and diameter) of this raindrop?

3. Consider the function $F(x) = e^x \sin(Nx)$ <u>on the interval</u> $[0, 1]$.

a) With a sketch or otherwise, describe this function when $N = 3$ and when $N = 10^{10}$.

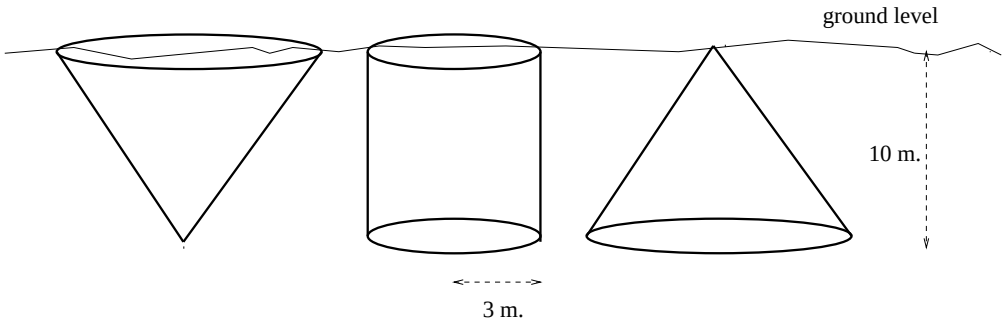
b) Compute $\int_0^1 F(x)\,dx$ if $N = 10^{10}$.

c) Explain b)'s result in relation to a).

4. Toxic waste (a homogeneous liquid whose density is 300 kg/m^3) is stored in three buried containers. Each container, illustrated below, is 10 meters tall. The top of each container is at ground level. Each container has the *same* volume. The middle container is a cylinder, and the other two are circular cones. Which container needs the *least* amount of work to empty (that is, to pump the toxic waste to ground level)? Which container needs the *most* work to empty? Justify your assertions by computing the work necessary in each case*.

OVER

* You probably should begin with the cylinder.



Toxic waste containers

Note The volume of a cylinder with base radius r and height h is $\pi r^2 h$; the volume of a cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.