November 19, 2005

Math 192

Review problems for the second exam

The real exam will certainly be much shorter than what's below. No calculators will be allowed. Definite integrals should be computed in terms of values of functions used in calculus and (where possible) named constants such as e and π. I will give out copies of the Math 152 second exam formula sheet with our exam. I will not ask about arc length or surface area. The exam will test material from lectures 10 and 13–20 on the Math 152 syllabus. That's essentially sections 9.1–9.3 and 11.1–11.9 of the textbook.

1. All of the following series converge. In each case, find a specific partial sum which is within .001 of the sum of the whole series. *Explain* why the specific partial sum you wrote satisfies this requirement. Please note that different partial sums may be needed for these series. You are *not* asked to find the "lowest" partial sum which satisfies this requirement.

a) $\sum_{n=1}^{\infty} \frac{5}{n^3}$ b) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$ c) $\sum_{n=1}^{\infty} \frac{(.4)^n}{(n!)^2}$

d) $\sum_{n=1}^{\infty} \frac{4^n}{(n!)^2}$ e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^4}$ f) $\sum_{n=100}^{\infty} \frac{(-1)^n}{\ln(\ln(\ln(n)))}$ *

2. The following series do not converge. For each, find a specific partial sum which is greater than 100, or indicate why this is not possible.

a) $\sum_{n=1}^{\infty} \frac{5}{n^{1/3}}$ b) $\sum_{n=1}^{\infty} (1.001)^n$ c) $\sum_{n=1}^{\infty} (-1)^n + 7\left(-\frac{1}{2}\right)^{3n+2} + 18\left(-\frac{1}{7}\right)^{5n+4}$

3. For which x's do these series converge? Where do they converge absolutely? Where do they converge conditionally?

a) $\sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdot 5\cdots(2n+1)} x^n$ b) $\sum_{n=0}^{\infty} \frac{x^n}{5\sqrt{n}}$

WARNING! The following series are *not* power series so their behavior may *not* be the same as power series!

c) $\sum_{n=1}^{\infty} n^2 e^{-nx}$ d) $\sum_{n=1}^{\infty} \frac{x^n}{1+x^{2n}}$

4. Suppose α and β are positive numbers. Decide whether the following sequences converge. If they converge, try to find their limits.† (Please give reasons for your assertions.)

a) The sequence $\{a_n\}$ with $a_n = \sqrt[n]{\alpha^n + \beta^n}$ b) The sequence $\{b_n\}$ with $b_n = \sqrt[n]{\alpha^n + \beta}$

5. Find all solutions of these differential equations.

a) $\frac{dy}{dx} = x^2y^2 + x^2$ b) $\frac{dy}{dx} = xe^y$

* You may wonder why this sum begins at n=100. Why does it?

† Your answers may involve both numbers and their relationship.

Comment The next two problems, slightly changed, are from exams at Rice University.

6. Consider the autonomous equation $y' = (y+1)(y^2-4)$.

a) Find and classify the equilibrium solutions. Sketch, on one set of axes, these solutions.*

b) On the same set of axes, sketch

i) A solution satisfying $y(0) = 3$.
ii) A solution satisfying $y(0) = 0$.
iii) A solution satisfying $y(1) = 0$.
iv) A solution satisfying $y(0) = -3$.

7. Consider the differential equation $y' = y^2 - y - ye^{-t}$.

a) Show that $y_1(t) = e^{-t}$ is a solution.

b) Find a solution of the form $y = \text{CONSTANT}$.

c) Sketch the solutions from parts a) and b) on the same set of axes.

d) Suppose that $y(t)$ is a solution of the differential equation which exists for all $t \geq 0$ for which $0 \leq y(0) \leq 1$. Verify that $y(t) \to 0$ as $t \to \infty$ with some brief written explanation.

8. a) Two students are sharing a loaf of bread. Student Alpha eats half of the loaf, then student Beta eats half of what's left, then Alpha eats half of what's left, and so on. How much of the loaf will each student eat?

b) Two students are sharing a loaf of bread. Student Alpha, now hungrier and more ferocious, eats two-thirds of the loaf, then student Beta eats eats half of what's left, then Alpha eats two-thirds of what's left, then Beta eats half of what's left, and so on. How much of the loaf will each student eat?

c) Now let's start with three students: Alpha, Beta, and Gamma. They decide to share a loaf of bread. Alpha eats half of the loaf, passes what's left on to Beta who eats half, and then on to Gamma who eats half, and then back to Alpha who eats half, and so on. How much of the loaf will each student eat?

9. True or false. If true, briefly indicate *why* the statement is true. If false, give a (simple) example to show that the statement is false.

a) $\left(\sum\limits_{n=0}^{\infty} a_n\right) \cdot \left(\sum\limits_{n=0}^{\infty} b_n\right) = \sum\limits_{n=0}^{\infty} a_n b_n$

b) If the sequence (x_n) converges, then the sequence $\left((x_n)^2\right)$ converges.

c) If the sequence $\left((x_n)^2\right)$ converges, then the sequence (x_n) converges.

10. a) Write $\int_0^2 \frac{x^2}{100-x^5}\,dx$ as the sum of a series.

b) Show that the function $f(x) = \sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ is a solution of the differential equation $f''(x) + f(x) = 0$.

c) Differentiate a geometric series and multiply by x twice to find the sum of $\sum\limits_{n=1}^{\infty} \frac{n^2}{5^n}$.

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* Added by me: which are stable and which are unstable?