October 12, 2005

Math 192

Review problems for the first exam

The real exam will certainly be much shorter than what's below. No calculators will be allowed. Definite integrals should be computed in terms of values of functions used in calculus and (where possible) named constants such as e and π.

1. Compute.

a) $\displaystyle\int_0^a x\sqrt{x^2+a^2}\,dx$ (here a is a positive constant)

b) $\displaystyle\int_0^\pi (\sin x)^5\ dx$

c) $\displaystyle\int_1^e x^2(\ln x)^2\ dx$

d) $\displaystyle\int_0^1 \frac{x^2-x+3}{(x+1)(x^2+1)}\,dx$

2. a) Sketch the area in the first quadrant enclosed by the curves $y=x^4$ and $x=y^2$.

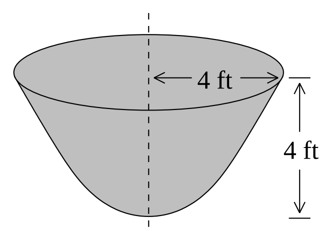
b) Compute this area.

c) Compute the volume which results when the area described above is revolved about the $x-$axis. Describe the method that you are using, and draw a sketch.

d) Compute the volume which results when the area described above is revolved about the $y-$axis. Describe the method that you are using, and draw a sketch.

3. The base of a solid is the ellipse $x^2+3y^2=1$. Cross-sections of the solid perpendicular to the $y-$axis are squares. What is the volume of the solid?

4. A tank full of water has the shape of a paraboloid of revolution as shown in the figure; that is, its shape is obtained by rotating a parabola about a vertical axis. If its height is 4 ft and the radius at the top is 4 ft, find the w3ork required to pump the water out of the tank. (Copied from p. 469 of the textboook)



5. Suppose $f(x)$ is a function with the following properties:

i) $f(0)=1$.

ii) The average value of the function $f(x)$ over any interval $[a,b]$ is $3\left(\dfrac{f(b)-f(a)}{b-a}\right)$.

Find a formula for $f(x)$.

7. Compute $\displaystyle\int_3^4 \frac{\sqrt{x^2-2}}{x}\,dx$ or $\displaystyle\int_{\frac{1}{2}}^1 \frac{x}{\sqrt{x^2-2}}\,dx$.

8. Compute $\displaystyle\int_3^4 \frac{1}{(x-1)^2(x+1)}\,dx$ and $\displaystyle\int_3^4 \frac{1}{(x-1)(x-2)(x+1)}\,dx$. Which is larger?

9. Consider the integral $\displaystyle\int_0^1 \sqrt{x+x^3}\,dx$. Use the theoretical error bound for the Trapezoidal Rule to find a value of n for which the error in the approximation is less than 10^{-4}.

10. a) Find a substitution u that shows $\int 2\,(\sec x)^2 \tan x \; dx = (\tan x)^2 + C$.

b) Find a different substitution u that shows $\int 2\,(\sec x)^2 \tan x \; dx = (\sec x)^2 + C$.

c) Can you conclude from a) and b) that the functions $(\sec x)^2$ and $(\tan x)^2$ are equal? If you cannot, describe and explain what you *can* conclude.

11. Evaluate $\int_0^b \sin(5x)\sin(8x)\,dx$ in terms of $\sin(5b)$, $\cos(5b)$, $\sin(8b)$, and $\cos(8b)$.

12. Find the antiderivatives:

a) $\int \cos(\log(x))\,dx$

b) $\int \frac{1}{\sqrt{1+e^x}}\,dx$

c) $\int \ln(a^2+x^2)\,dx$

d) $\int \sin\left(\sqrt{x+1}\right)\,dx$

13. Determine whether each of the following is convergent or divergent, and evaluate those that are convergent. Be sure to show your work and explain your reasoning.

a) $\int_1^e \frac{1}{x\,(\ln x)^2}\,dx$

b) $\int_e^\infty \frac{1}{x\,(\ln x)^2}\,dx$

c) $\int_2^4 \frac{1}{\sqrt{x-2}}\,dx$

d) $\int_0^{\pi/4} \frac{1}{(x^2+1)\,\arctan x}\,dx$

14. `Maple` can find an explicit antiderivative of $\frac{1}{\sqrt{x}+x^2}$ but the result is quite complicated (it has 9 different terms!). Please show that $\int_0^\infty \frac{1}{\sqrt{x}+x^2}\,dx$ converges and find some convenient overestimate for its value. Explain why your estimate is valid.