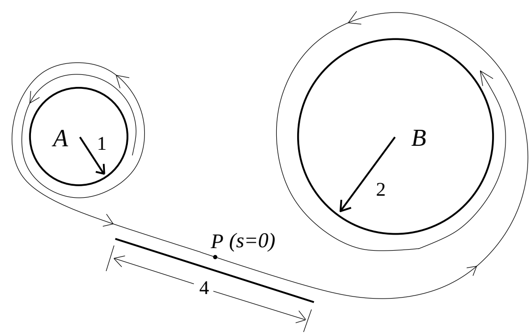
(8) 1. Suppose $p = (1,0,2)$, $q = (0,2,2)$, and $r = (1,1,1)$ are points in $\mathbb{R}^3$.
a) Find a vector orthogonal to the plane through the points p, q, and r.
b) Find the area of triangle pqr.

(12) 2. Suppose $\mathbf{V} = \langle 1,3,-2\rangle$ and $\mathbf{W} = \langle -1,2,1\rangle$. Find vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ so that $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$, $\mathbf{V}_1$ is parallel to $\mathbf{W}$, and $\mathbf{V}_2$ is perpendicular to $\mathbf{W}$.

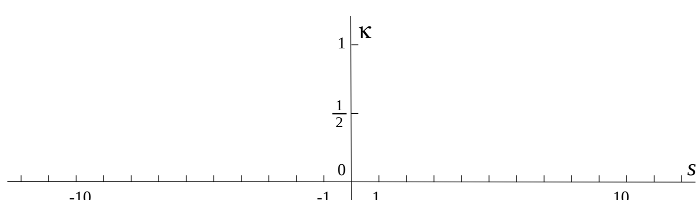
(10) 3. The point $(1,0,2)$ is on the plane P whose equation is given by $2x - 2y + z = 4$.
a) Find parametric equations for the line perpendicular to P through $(1,0,2)$.
b) Find a point on the line in a) which has distance 5 to P.
c) Write an equation for a plane parallel to P whose distance to P is 5.

(12) 4. Suppose the position vector of a curve is given by $\mathbf{r}(t) = \langle e^t, e^{-t}, \sqrt{2}t\rangle$. Find the unit tangent and unit normal vectors when the parameter $t = 1$. That is, find $\mathbf{T}(1)$ and $\mathbf{N}(1)$.

(12) 5. A point is moving along the curve below in the direction indicated. Its motion is parameterized by arc length, s, so that it is moving at unit speed. Arc length is measured from the point P (both backward and forward). The curve is intended to continue indefinitely both forward and backward in s, with its forward motion curling more and more tightly around the indicated circle, B, and, backward, curling more and more tightly around the other circle, A. Near P the curve is parallel to the indicated line segment.



Sketch a graph of the curvature, κ, as a function of the arc length, s. What are $\lim_{s\to+\infty} \kappa(s)$ and $\lim_{s\to-\infty} \kappa(s)$? Use complete English sentences to briefly explain the numbers you give.
Note that the units on the horizontal and vertical axes differ in length.



$\lim_{s\to-\infty} \kappa(s) =$ ______ $\lim_{s\to+\infty} \kappa(s) =$ ______

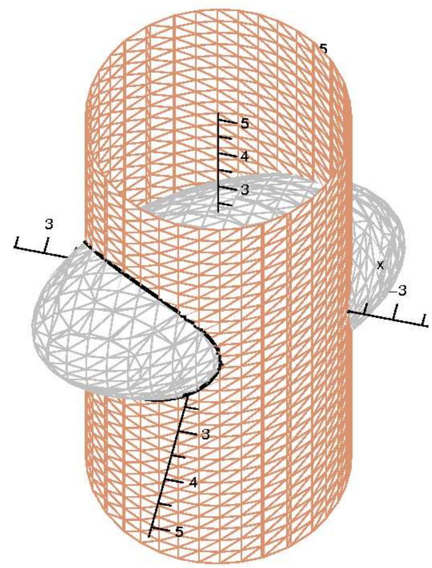
(10) 6. Find the linear approximation of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at $(2,1)$ and use it to approximate $f(1.95, 1.08)$.

(12) 7. a) If $z = f(x^2 - 3y)$, show that $3\dfrac{\partial z}{\partial x} + 2x\dfrac{\partial z}{\partial y} = 0$.

b) If $z = f(x^2 - 3y)$, then $\dfrac{\partial^2 z}{\partial x^2} = A(x)f''(x^2 - 3y) + B(x)f'(x^2 - 3y)$ where $A(x)$ and $B(x)$ are simple functions of x alone or constants. What are $A(x)$ and $B(x)$?

(12) 8. Suppose that $F(x, y, z) = x^2 + 3yz$ and $p = (-3, 2, -1)$.
a) Find the maximum directional derivative of F at p and write a unit vector pointing in the direction this maximum value occurs.
b) Suppose $C = F(-3, 2, -1)$. Compute C and write an equation for the plane tangent to the surface $F(x, y, z) = C$ at the point p.

(12) 9. a) If $f(x, y, z) = x^2 + y^2$, compute $\nabla f(x, y, z)$. What are $f(2, 1, 2)$ and $\nabla f(2, 1, 2)$?
b) If $g(x, y, z) = x^2 + y^2 + z^2 - xy - yz$, compute $\nabla g(x, y, z)$. What are $g(2, 1, 2)$ and $\nabla g(2, 1, 2)$?
c) The point $(2, 1, 2)$ is on both the surface $x^2 + y^2 = 5$, a circular cylinder whose axis of symmetry is the z-axis, and the surface $x^2 + y^2 + z^2 - xy - yz = 5$, an ellipsoid tilted with respect to the coordinate axes. The surfaces intersect in a curve. The surfaces and the curve are shown in the picture to the right. Find a vector tangent to that curve at $(2, 1, 2)$. Your answers to a) and b) can be used here.



A A

First Exam for Math 251, sections 5–10

February 25, 2006

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	8	
2	12	
3	10	
4	12	
5	12	
6	10	
7	12	
8	12	
9	12	
Total Points Earned:		

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