Useful information for the second exam in Math 251:05-10, spring 2006

The second exam will be given Friday, April 7. The place and time will be your usual class time and class location.

The exam will cover the material of the syllabus up to and including lecture 20 (that's section 16.3 of the text). Some material done before the first exam will certainly be tested (for example, you'd better know how to manipulate vectors, take partial derivatives, and use the chain rule) but the emphasis will be on what's been done since then. I did not cover section 15.9 so the material from that section will not be on the exam. I have not systematically covered section 15.5, so no specific formulas or ideas from that section will be needed on this exam. Some exam rules and suggestions:

• No books or notes. A formula sheet will be handed out with the exam. A *draft version* of the formula sheet will be linked to the course webpage.
• No calculators of any kind may be used during the exam. Please leave answers in "unsimplified" form – so $15^2 + (.07) \cdot (93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Traditional math constants such as π and e should be left "as is" and not approximated.
• Show your work: an answer alone may not receive full credit. Write clearly; label answers.

Here are some problems from past exams. There are almost three times as many problems as a "real" exam would have.

A Sketch the region described in spherical coordinates by $0 \le \phi \le \pi/3$ and $0 \le \rho\cos\phi \le 2$ and $0 \le \theta \le 2\pi$.

B Let $f(x,y) = \frac{1}{x} + \frac{1}{y} + xy$. True or false? Explain BRIEFLY:
a) f has a local maximum at $(1,1)$. b) f has a saddle point at $(1,-1)$.

C Find the absolute maximum and minimum of $f(x,y) = x^2 + 2x + 2y^2$
a) on the circle $x^2 + y^2 = 4$; b) on the disk $x^2 + y^2 \le 4$.

D Compute $\iiint_B (x^2 + y^2)\, dV$ if B is the "eastern hemisphere" $x \ge 0$ of the unit ball.

E Find all the critical points of $f(x,y) = x^2 - 4xy + y^3 - 3y$ and describe the type of each critical point.

F Find the maximum and minimum values of $x - 4y + 2z$ subject to the constraint that (x,y,z) lies on the ellipsoid $x^2 + y^2 + 5z^2 = 1$.

G Change the order of integration in $\int_1^2 \int_0^{\ln y} f(x,y)\, dx\, dy$.

H Change the integral $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2 + y^2 + z^2)\, dz\, dx\, dy$ to spherical coordinates. DON'T EVALUATE THE INTEGRAL.

I a) Compute $\int_2^3 \int_{1/x}^{x^2} x^2 y - 2x\; dy\, dx$.
b) Write this iterated integral in "$dx\, dy$" order. You may want to begin by sketching the area over which the double integral is evaluated. You are **not** asked to evaluate the $dx\, dy$ result, which may be one or more iterated integrals.

J Find the maximum and minimum of the function $f(x,y) = x^2 y$ subject to the constraint $x^2 + 2y^2 = 6$.

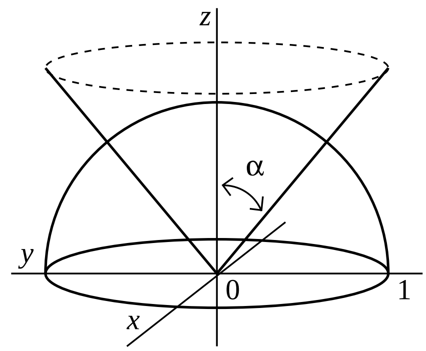
K Calculate $\iint_D y\, dA$ where D is bounded by $y = x - 1$ and $y^2 = 2x + 6$.

L Let $\mathbf{F}(x,y) = e^{2y}\mathbf{i} + (1 + 2xe^{2y})\mathbf{j}$. Find a function $f(x,y)$ such that $\nabla f = \mathbf{F}$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{r}(t) = \sqrt{t}\mathbf{i} + (1 + t^3)\mathbf{j}$, $0 \le t \le 1$.

M Set up a triple integral to find the volume of the solid bounded by the cylinder $x = y^2$ and the planes $z = 0$ and $x + z = 1$. DON'T EVALUATE THIS INTEGRAL.

N Compute the volume of the solid bounded by the xz-plane, the yz-plane, the xy-plane, the planes $x = 1$ and $y = 1$, and the surface $z = x^2 + y^4$.

O In this problem H is the upper half of the unit sphere in $\mathbb{R}^3$: those (x,y,z) with $x^2 + y^2 + z^2 \le 1$ and $z \ge 0$. There is a right circular cone whose vertex is $(0,0,0)$ and whose axis of symmetry is the positive z-axis which divides the volume of H into two equal parts. Find the angle α that determines this cone. The diagram defines α, which is the angle that the positive z-axis makes with a line on the cone through the vertex.



P Suppose D is the path consisting of three straight line segments, first from $(1,2)$ to $(4,-3)$, then from $(4,-3)$ to $(2,6)$, and then from $(2,6)$ to $(3,4)$. Compute $\int_D (2xy^3)\,dx + (3x^2y^2 + 4y^3)\,dy$.

Q Sketch the three level curves of the function $W(x,y) = ye^x$ which pass through the points $P = (0,2)$ and $Q = (2,0)$ and $R = (1,-1)$. **Label each curve** with the **appropriate function value**. Be sure that your drawing is clear and unambiguous.
Also, sketch on the same axes the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S = (0,-2)$ and the point $T = (-2,0)$.

R Compute the triple integral of the function $e^{-(x^2+y^2)}$ over the region in the first octant of $\mathbb{R}^3$ which is under the paraboloid $z = 1 - (x^2 + y^2)$. (The *first octant* in (x,y,z)−space is the collection of points which have $x \ge 0$, $y \ge 0$, and $z \ge 0$.)
Note Be careful: π and e both *definitely* appear in the answer!

S Let C be the straight line segment from $(0,1)$ to $(4,3)$. Find $\int_C x^2y\,ds$.

T Compute $\iint_R x\,dA$, where R is the region to the right of the y-axis and bounded by the circle of radius 2 centered at the origin, the positive part of the y−axis and the line $y = -x$.

U Evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\,dy\,dx$ by reversing the order of integration.

V Suppose $f(x,y,z) = xy^2z^3$.
a) Compute $\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx$.
b) Write the integral in a) as a sum of one or more iterated integrals in $dx\,dy\,dz$ order. You are *not* asked to integrate your answer, only to set it up.

W The average value of a function F defined in a region $\mathcal{R}$ of $\mathbb{R}^3$ is $\dfrac{\iiint_{\mathcal{R}} f\,dV}{\iiint_{\mathcal{R}} dV}$. Compute the average distance to the center of a sphere of radius a.

X Evaluate the triple integral $\iiint_E xy\,dV$ where E is bounded by the planes $x = 0$, $y = 0$, $z = 0$, $y + z = 1$ and $x + z = 1$.