Math 251:05-10 # Workshop Problems #2 **February 15, 2006**

1. Suppose $h(x, y) = x^y$. Find the domain of h and the first partial derivatives of h. I know that $2^3 = 8$. If x is increased by .01 (so (x, y) changes from $(2, 3)$ to $(2.01, 3)$), approximate the change in h using the linear approximation. If y is increased by .01 (so (x, y) changes from $(2, 3)$ to $(2, 3.01)$), approximate the change in h using the linear approximation. Compare the "exact answers" to the linearization answers. Which change increases the value of h more?

2. A certain function $f(x, y)$ is known to have partial derivatives of the form

$$\frac{\partial f}{\partial x} = 2y\cos(2x) + y^3x^2 + g(y) \text{ and } \frac{\partial f}{\partial y} = \sin(2x) + x^3y^2 + 4x + 1. \qquad (*)$$

Please note that g is a function of y only. Use the equality of mixed partial derivatives (Clairaut's Theorem) to find the function g up to an arbitrary additive constant. Then find all functions f with partial derivatives of the form $(*)$.

3. a) Suppose $f(u, v)$ is a differentiable function of u and v, and that $u = e^x \cos y$ and $v = e^x \sin y$. Express $\frac{\partial f}{\partial x}$ in terms of $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial v}$, and functions involving x and y.

b) Note that if $x = 0$ and $y = \frac{\pi}{2}$ then $u = 0$ and $v = 1$. Suppose also that you know $\frac{\partial f}{\partial u}(0, 1) = 7$ and $\frac{\partial f}{\partial v}(0, 1) = -3$. Use this information together with your formula in a) to compute $\frac{\partial f}{\partial x}$ when $x = 0$ and $y = \frac{\pi}{2}$.

4. A rectangular box with an open top has a square base. The sides are made of cardboard, costing 3 cents per square foot. The base is made of plywood, costing a half dollar per square foot. The box should have a capacity of no more than 10 cubic feet and no less than 2 cubic feet. At the same time, due to limitations of construction, no edge of the box should be shorter than 3 inches or longer than 36 inches. Find a plausible domain for the dimensions of the box based on these specifications and describe the domain carefully, algebraically. Sketch the domain in $\mathbb{R}^2$. (You *must* give a complete algebraic description of the domain, however. The picture is *not* a substitute for this description.) Write a formula for a function which calculates the cost of the materials in each possible box.

Grading workshop problems

Your recitation instructor will indicate one of these problems whose solution is requested in one week (at the Wednesday, February 22, meeting of your recitation section). Your workshop writeup will be read either by the lecturer or the recitation instructor. Grading will be on a 10 point scale: 5 points for mathematical content and 5 points for exposition. Further explanation of what is desired will be linked to the course webpage.