Math 251:05-10 **Workshop Problems #3** **March 6, 2006**

1. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate (including the boundary $x^2 + y^2 = 1$) is heated so that the temperature T at any point (x, y) is given by $T(x, y) = x^3 - x + 2y^2$. Locate the hottest and coldest points of the plate and determine the temperature at each of those points.

2. a) A rectangle with length L and width W is cut into four smaller rectangles by two lines which are parallel to the sides. Find the minimum value of the sum of the squares of the areas of the smaller rectangles.

b) Show that the maximum of the sums of the squares of the areas occurs when the cutting lines correspond to sides of the rectangle (so that there is only one rectangle).

3. Suppose R is the unit square where $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Does $\displaystyle\iint_R \frac{1}{x+y}\, dA$ converge*, and, if it does, what is its value? Include a simple sketch of the volume indicated in this integral.

4. Some of these iterated integrals correspond to real geometric problems (computation of volumes) and some do not. Some are actually illegal! Please indicate which are good and which are not. Explain your answers. Compute any iterated integral which corresponds to a volume. Include a sketch of each volume computed.

a) $\displaystyle\int_0^{x^3}\int_0^{y} x^4 + y^2 + 7\, dx\, dy$

b) $\displaystyle\int_0^{1}\int_0^{5x} x^4 + y^2 + 7\, dx\, dy$

c) $\displaystyle\int_5^{7}\int_{y^3}^{3y} x^4 + y^2 + 7\, dx\, dy$

d) $\displaystyle\int_{-1}^{0}\int_{-y^2}^{2y} x^4 + y^2 + 7\, dx\, dy$

5. Compute:

a) $\displaystyle\int_0^{\frac{\pi}{2}} \left(\int_y^{\frac{\pi}{2}} \frac{\sin x}{x}\, dx \right) dy$ b) $\displaystyle\int_0^{1} \left(\int_{\sqrt{y}}^{1} e^{(7x^3)}\, dx \right) dy$ c) $\displaystyle\int_0^{1} \left(\int_x^{x^{1/3}} \sqrt{1 - y^4}\, dy \right) dx$

Hint: write 'em as a double integral, then re-iterate**.

Grading workshop problems

Your recitation instructor will indicate one of these problems whose solution is requested in one week (at the Wednesday, March 22, meeting of your recitation section). Your workshop writeup will be read either by the lecturer or the recitation instructor. Grading will be on a 10 point scale: 5 points for mathematical content and 5 points for exposition. Further explanation of what is desired will be linked to the course webpage.

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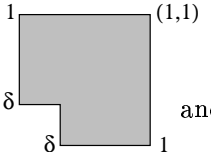
* The integral is improper. Use limits: first integrate over 

and then let the notch sides $\to 0^+$.

** Yes, this is supposed to be a somewhat incomprehensible clue.