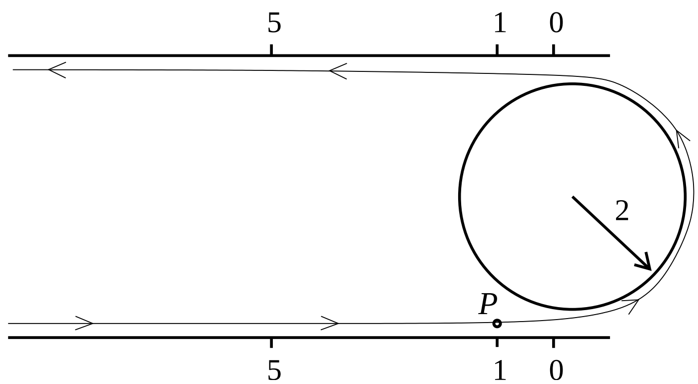
1A

(8) 1. Suppose the position vector of a curve is given by $\mathbf{r}(t) = \langle \sin(2t), \cos(3t), t^2 \rangle$. Find the unit tangent vector to this curve when $t = \pi$. That is, find $\mathbf{T}(\pi)$.

(8) 2. Find a vector parameterization for the line which passes through $(1, 1, 1)$ and $(3, -5, 2)$.

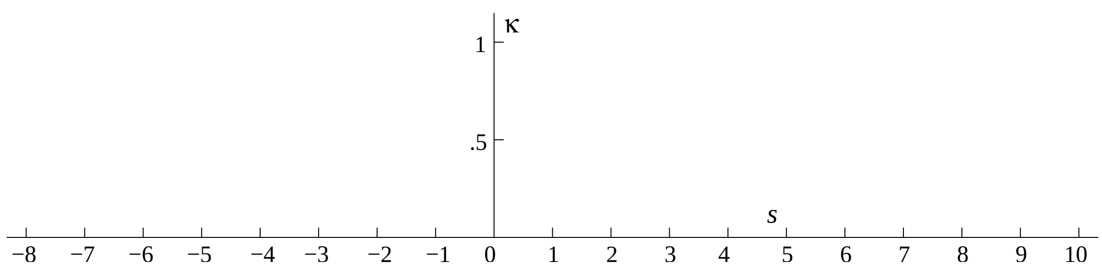
12.2 #31

(8) 3. A point moves along the curve drawn below in the direction indicated. Its motion is parameterized by arc length, s, so that it is moving at unit speed. Arc length is measured from the point P (both backward and forward). The curve is intended to continue indefinitely both forward and backward in s, with its forward motion coming closer and closer to the indicated straight line, and backward, coming closer and closer to the other indicated straight line. The numbers on each line indicate distance along that line.



Sketch a graph of the curvature, κ, as a function of the arc length, s.

What are $\lim_{s \to +\infty} \kappa(s)$ and $\lim_{s \to -\infty} \kappa(s)$?



Note that the units on the horizontal and vertical axes differ in length.

$\lim_{s \to -\infty} \kappa(s) =$ ______ $\lim_{s \to +\infty} \kappa(s) =$ ______

(8) 4. Find two vectors (which are not multiples of each other) that are both orthogonal to $\langle 2, 0, -3 \rangle$.

12.3 #31

(8) 5. Calculate the cross product: $(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - 7\mathbf{k})$.

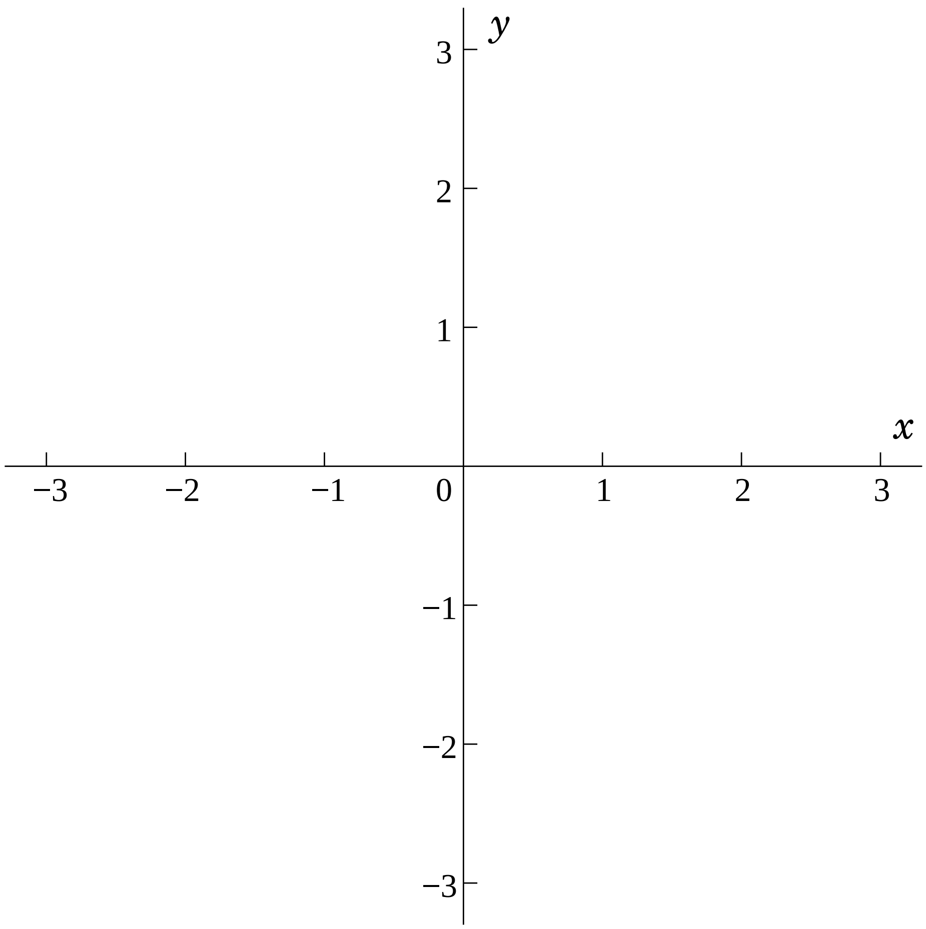
12.4 #20

(8) 6. A bee with velocity vector $\mathbf{r}'(t)$ starts out at the origin at $t = 0$ and flies around for T seconds. Where is the bee located at time T if $\int_0^T \mathbf{r}'(u)\,du = 0$? What does the quantity $\int_0^T \|\mathbf{r}'(u)\|\,du$ represent?

13.3 #13

(10) 7. Sketch two level curves of $f(x,y) = \dfrac{x-y}{x+y}$ and *label* these curves appropriately. Explain why $\lim_{(x,y)\to(0,0)} \dfrac{x-y}{x+y}$ does or does not exist.

Review question A



Does the limit exist? **YES** **NO** (Circle exactly one of these.)

Brief explanation of your answer

(10) 8. If $h(x,z) = e^{2xz - x^2z^3}$, compute $\dfrac{\partial^2 h}{\partial x \partial z}$ when $x = 2$ and $z = 1$.

Like 14.3 #43

(8) 9. Suppose $g(w)$ is any differentiable function of one variable. If $f(x, y) = g(x^2 - 3y)$, verify that $3\dfrac{\partial f}{\partial x} + 2x\dfrac{\partial f}{\partial y} = 0$.

Part of problem 7 on an old exam

(12) 10. a) Find an equation of the tangent plane to the surface at the given point:

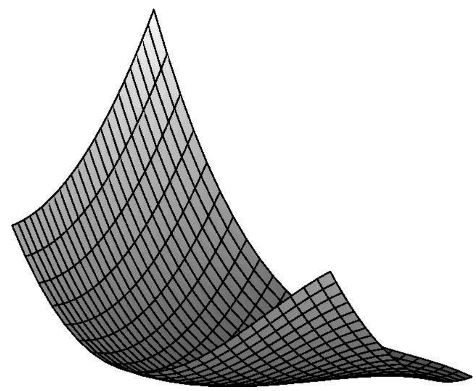
$$xz + 2x^2y + y^2z^3 = 11,\ P = (2, 1, 1).$$

14.5 #42

b) If $f(x, y, z) = xz + 2x^2y + y^2z^3$, find a unit vector in the direction of the largest directional derivative of f at $P = (2, 1, 1)$. What is the value of that directional derivative? (Information obtained in a) would allow you to answer these questions immediately.)

(12) 11. Locate and identify (max, min, saddle) all the critical points of $f(x, y) = \dfrac{x^3}{3} + \dfrac{y^2}{2} - xy$.

Comment To the right is a picture of part of the graph of $z = f(x, y)$ which contains the critical points. This may help, but your solution *must* be algebraic.



A **A**

First Exam for Math 251, sections 22–24

October 15, 2008

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes and no calculators may be used on this exam.

"Simplification" of answers is not necessary,
but standard values of traditional functions
such as e^0 and $\sin\left(\frac{\pi}{2}\right)$ should be given.

Problem Number	Possible Points	Points Earned:
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	10	
8	10	
9	8	
10	12	
11	12	
Total Points Earned:		

A **A**