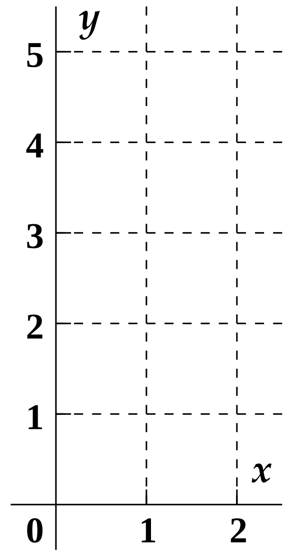
(12) 1. Theoretical results imply that $x + 3yz$ has a maximum and a minimum on the sphere $x^2 + y^2 + z^2 = 1$. Use Lagrange multipliers to find these maximum and minimum values.

(12) 2. Suppose $I = \int_0^2 \int_{x^2}^5 xy\,dy\,dx$.

a) Compute I.

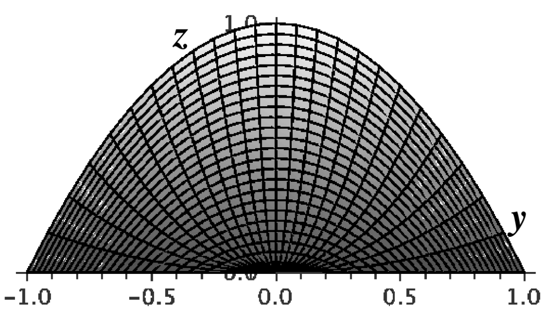
b) Use the axes to the right to sketch the region of integration for I.

c) Write I as a sum of one or more $dx\,dy$ integrals. You do not need to compute the result!

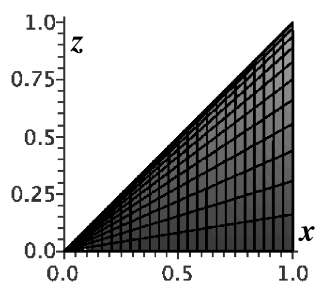


(12) 3. The coordinates (x, y, z) of points in a solid object A in $\mathbb{R}^3$ satisfy the inequalities $0 \le z \le x - y^2$ and $0 \le x \le 1$. Compute the triple integral of 1 over the object A. (This is the volume of A.)

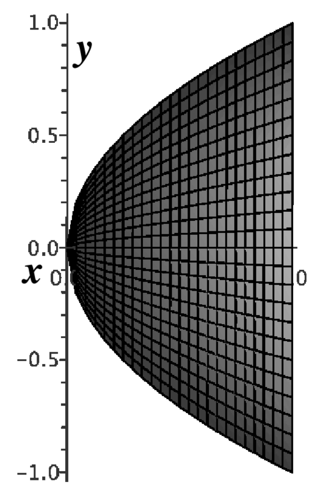
Below are some pictures of the object which may be helpful.



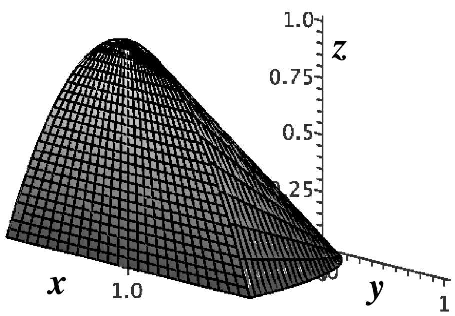
View from the x-axis; the z-axis is up and the y-axis is horizontal.



View from the y-axis; the z-axis is up and the x-axis is horizontal.



View from the z-axis; the y-axis is up and the x-axis is horizontal.



Oblique view; the z-axis is up, the x-axis is to the left and the y-axis is to the right.

(12) 4. Compute $\displaystyle\iint_D e^{-x^2-y^2}\,dA$ where D is the region in the plane which is inside the unit circle (the circle with center at $(0,0)$ and radius 1) and also inside the upper half plane (where $y \geq 0$).

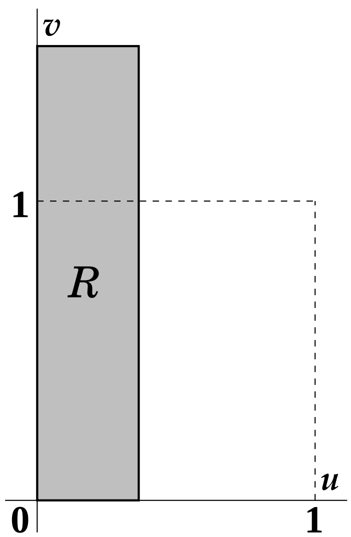
(12) 5. Express in cylindrical coordinates and evaluate: $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} z\,dz\,dy\,dx\,.$

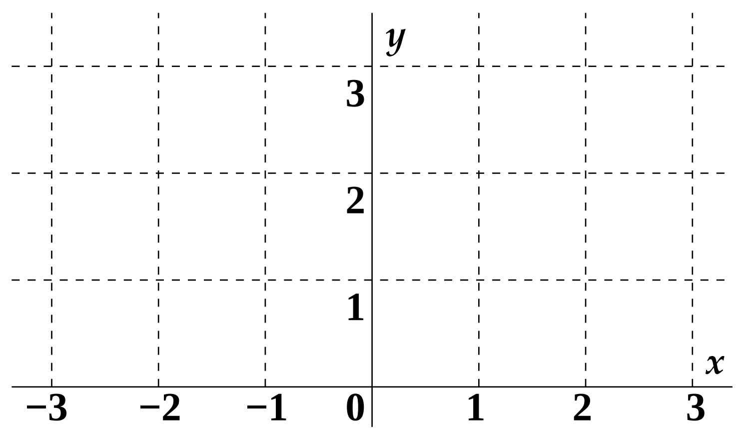
(12) 6. Use spherical coordinates to calculate the triple integral of $f(x,y,z) = x^2+y^2+z^2$ over the region $1 \leq x^2+y^2+z^2 \leq 4\,.$

(12) 7. This problem is about the transformation $\begin{cases} x = e^{3u}\cos(2v) \\ y = e^{3u}\sin(2v) \end{cases}.$

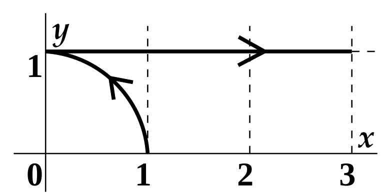
a) Compute the Jacobian of this transformation. The result should be $6e^{6u}$ but you must show the details of the computation.

b) Suppose R is the region in the uv-plane determined by $u=0$, $u=\frac{1}{3}$, $v=0$, and $v=\frac{\pi}{2}$ as shown on the coordinate axes below and to the left. Sketch the image region using this transformation in the xy-plane below and to the right.





(16) 8. a) Compute $\displaystyle\int_C x\,dx + y^2\,dy$ if C is a quarter circle centered at $(0,0)$ from $(1,0)$ to $(0,1)$ followed by a line segment from $(0,1)$ to $(3,1)$.



C is shown in a diagram to the right. You may need more than one integral!

b) Suppose $\mathbf{F}$ is the vector field $(x+5y^2)\mathbf{i} + (Axy)\mathbf{j}$, where A is a constant. There is one value of A for which this vector field is a gradient vector field. Find that value of A. Then find all potentials of $\mathbf{F}$, using that value of A.

A **A**

Second Exam for Math 251, sections 22–24

November 19, 2008

NAME ______________________________

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes and no calculators may be used on this exam.

"Simplification" of answers is not necessary, but standard values of traditional functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$ should be given.

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	16	
Total Points Earned:		

A **A**