Just a few formulas for the final exam in Math 251, fall 2008

Curvature κ is all of the following:

$$\left\|\frac{d\mathbf{T}}{ds}\right\| = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3} \overset{2\ \text{dim}}{=} \frac{|y''(t)x'(t) - x''(t)y'(t)|}{(x'(t)^2+y'(t)^2)^{3/2}} \overset{y=f(x)}{=} \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}$$

Second derivative test for differentiable functions in $\mathbb{R}^2$

Suppose $f_x(a,b)=0$ and $f_y(a,b)=0$. Let $H = H(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.

a) If $H>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local minimum.

b) If $H>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local maximum.

c) If $H<0$, then $f(a,b)$ is not a local maximum or minimum (f has a saddle point).

If $H=0$, no information.

Polar coordinates $dA = r\,dr\,d\theta$

Spherical coordinates $dV = \rho^2\sin\phi\,d\rho\,d\theta\,d\phi$

Change of variables in 2 dimensions

$$\iint_{R_{xy}} f(x,y)\ dA = \iint_{\tilde{R}_{uv}} f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$$ where the Jacobian, $\frac{\partial(x,y)}{\partial(u,v)}$ is $\det\begin{pmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{pmatrix}$.

Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

If $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$ and $\mathbf{F}$ is a vector field then $\begin{cases}\text{curl } F = \nabla\times\mathbf{F}, & \text{a vector field.}\\ \text{div } F = \nabla\cdot\mathbf{F}, & \text{a function.}\end{cases}$

Stokes' Theorem

S is a surface with boundary curve C. As you "walk" along C, S is to the left and $\mathbf{N}$, the surface normal, is up.

$$\left[\iint_S (\text{curl }\mathbf{F})\cdot\mathbf{N}\,dS = \right]\quad \iint_S (\nabla\times\mathbf{F})\cdot d\mathbf{S} = \int_C \mathbf{F}\cdot d\mathbf{s}\quad \left[= \int_C P\,dx + Q\,dy + R\,dz\right]$$

Divergence Theorem

W is a region in $\mathbb{R}^3$ with boundary surface S. The boundary S is oriented so its normal vectors point *outward.*

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iiint_E \text{div } F\,dV \quad \left[= \iiint_E \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}\,dV\right]$$