CD 1

Suppose the equations for the planes P_1 and P_2 are $2x - 2y + 3z = 3$ and $6x + y - 2z = 5$, respectively, where P_1 and P_2 are not parallel.

a) By definition, the angle between two planes is the angle between their normal vectors. The normal vectors for P_1 and P_2 are $\langle 2, -2, 3 \rangle$ and $\langle 6, 1, -2 \rangle$, respectively. The dot product and the angle Θ between nonzero vectors are related by

$$\cos(\Theta) = \frac{\langle 2, -2, 3 \rangle \cdot \langle 6, 1, -2 \rangle}{\| \langle 2, -2, 3 \rangle \| \| \langle 6, 1, -2 \rangle \|} = \frac{2(6) + 1(-2) + 3(-2)}{\sqrt{(17)} \cdot \sqrt{(41)}}$$

$$\Rightarrow \quad \Theta = \arccos(4 / \sqrt{(697)}).$$

b) Just by looking at the equations for each plane, it appears that the point (1,1,1) is on both P_1 and P_2. To check...

P_1) $2(1) - 2(1) + 3(1) = 2 - 2 + 3 = 3$
P_2) $6(1) + (1) - 2(1) = 6 + 1 - 2 = 5$

c) We already have a point, (1,1,1), so we only need a direction vector to parameterize the line of intersection of P_1 and P_2. The cross product of their normal vectors will give us this direction vector.

$$\langle 2, -2, 3 \rangle \times \langle 6, 1, -2 \rangle = \det \begin{matrix} i & j & k \\ 2 & -2 & 3 \\ 6 & 1 & -2 \end{matrix}$$

$= ((-2)(-2) - 3(1)) \cdot i \; - \; (2(-2) - 3(6)) \cdot j \; + \; (2(1) - (-2)(6)) \cdot k$
$= 1 \cdot i \; + \; 22 \cdot k \; + \; 14 \cdot k \; = \; \langle 1, 22, 14 \rangle$

Now we have both a point and a direction, so we can parameterize the line of intersection of P_1 and P_2 by...

$$c(t) = (t + 1 , 22t + 1 , 14t + 1)$$