(20) 1. In this problem, suppose that $f(x,y,z) = \dfrac{x^3 - 2yz}{y^2 + xz}$. Notice that $f(-1,1,2) = 5$.
a) Find $\nabla f(x,y,z)$. You do **not** need to "simplify" your answer! Compute $\nabla f(-1,1,2)$ which you may wish to simplify.
b) Write an equation for the plane tangent to $f(x,y,z) = 5$ at the point $(-1,1,2)$. You do **not** need to "simplify" your answer!
c) Write parametric equations for the line normal to $f(x,y,z) = 5$ at the point $(-1,1,2)$. You do **not** need to "simplify" your answer!
d) Find the directional derivative of f in the direction of the unit vector $\langle -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle$ at the point $(-1,1,2)$. You do **not** need to "simplify" your answer!
e) Find a unit vector in the direction of the largest directional derivative of f at the point $(-1,1,2)$. You do **not** need to "simplify" your answer!
f) What is the value of the largest directional derivative of f at the point $(-1,1,2)$? You do **not** need to "simplify" your answer!

(14) 2. Suppose that $x^2 + px + q$ has roots r and s.

$$\boxed{x^2 + x - 6 = (x-2)(x+3)}$$

a) Write formulas for r and s as functions of p and q. (Nothing more is asked here: only "high school algebra".)
b) Verify that the functions found in a) give 2 and -3 for r and s if $p = 1$ and $q = -6$.
c) Suppose p changes from 1 to 1.03 and q, from -6 to -6.04. Use linear approximation applied to the functions found in a) to find the approximate changes in the roots r and s. You do **not** need to "simplify" your answer!

(16) 3. a) Find an equation of the plane through $(4,1,-2)$ which contains the line $\mathbf{r}(t) = \langle 4,1,6\rangle + t\langle 1,4,1\rangle$.
b) The plane found in a) and the line $\mathbf{s}(t) = \langle -2,0,3\rangle + t\langle 3,1,1\rangle$ intersect. Find the point of intersection. You do **not** need to "simplify" your answer!

(12) 4. If $x = s^2 - t^2$, $y = 2st$, and $z = f(x,y)$, show that $\left(\frac{\partial z}{\partial s}\right)^2 + \left(\frac{\partial z}{\partial t}\right)^2 = 4\sqrt{x^2+y^2}\left(\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right)$.

(8) 5. Find $\lim_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$ or show that the limit does not exist.

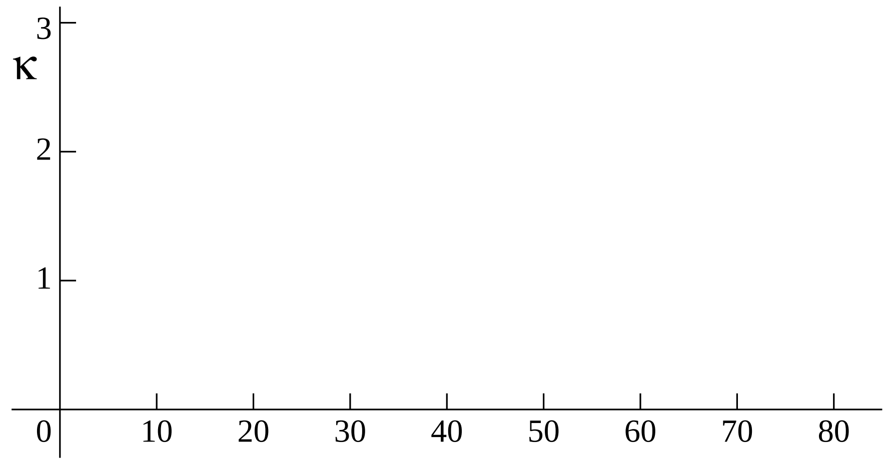
(12) 6. Suppose the function $f(x,y)$ with domain all of $\mathbb{R}^2$ is defined by $f(x,y) = \begin{cases} y & \text{if } y > x^2 \\ x & \text{if } y \le x^2 \end{cases}$.
a) Sketch a graph of $z = f(x,y)$. (You may wish to sketch two graphs and assert that your answer is a combination of these two!)
b) For which points (x,y) is $f(x,y)$ continuous? Consider all possible points in the domain, $\mathbb{R}^2$. Give some explanations for your answers.

(10) 7. A particle has position vector given by $\mathbf{R}(t) = \frac{1}{t}\mathbf{i} + t^2\mathbf{j} - 3t\mathbf{k}$.
a) What are the velocity and acceleration vectors of this particle when $t = 1$?
b) Write the acceleration vector when $t = 1$ as a sum of two vectors, one parallel to the velocity vector when $t = 1$ and one perpendicular to the velocity vector when $t = 1$.

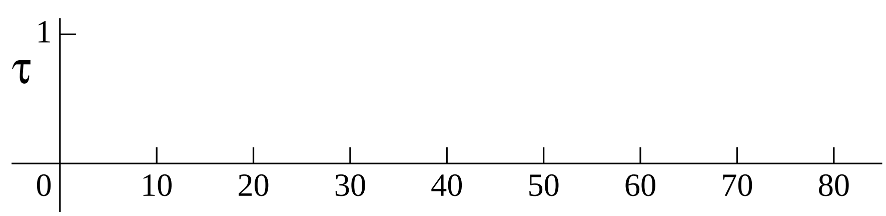
(8) 8. The flight of an airplane is described in this paragraph:

A The plane flies straight north for 30 miles.
B The plane then makes a level quarter circular turn of radius $\frac{1}{3}$ mile. There is no change in altitude.
C The plane then flies straight east for 20 miles.
D The plane then gains altitude, flying on a right circular helical curve which has base radius 2 miles. The plane flies one and half loops of the helix and has a 5 mile increase in altitude.
E The plane then flies straight west for 10 miles.

a) Sketch a graph of the curvature, κ, of the plane flight as a function of the distance the plane has traveled. Write on the horizontal axis the letters **A**, **B**, **C**, **D**, and **E** when the plane is beginning the part of the flight corresponding to the description above. The graph should be *qualitatively* correct. Although exact numerical results are *not* needed, the vertical axis shown is probably sufficient to answer the question completely.



b) Sketch a graph of the torsion, τ, of the plane flight as a function of the distance the plane has traveled. Write on the horizontal axis the letters **A**, **B**, **C**, **D**, and **E** when the plane is beginning the part of the flight corresponding to the description above. The graph should be *qualitatively* correct. Although exact numerical results are *not* needed, the vertical axis shown is probably sufficient to answer the question completely.



First Exam for Math 291, section 1

October 19, 2006

NAME ____

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes may be used on this exam other than the formula sheet to be distributed.

Problem Number	Possible Points	Points Earned:
1	20	
2	14	
3	16	
4	12	
5	8	
6	12	
7	10	
8	8	
Total Points Earned:		