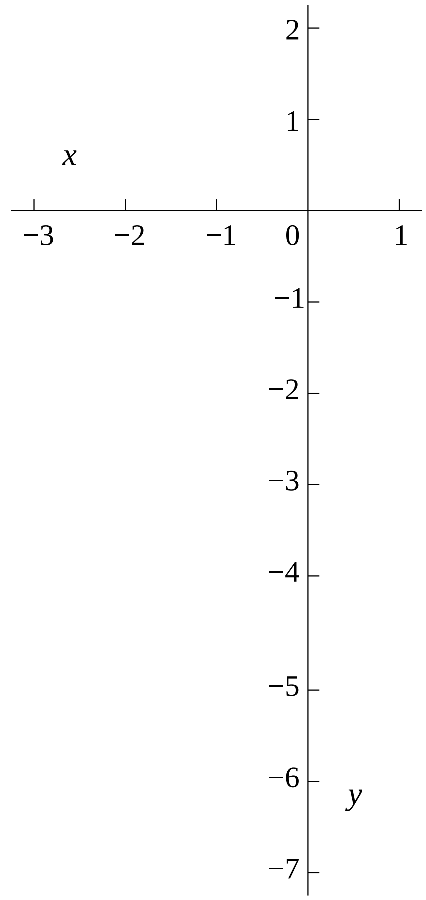
(15) 1. This problem analyzes $\int_{-3}^{1}\int_{2x-1}^{2-x^2} x^2\,dy\,dx$.

a) Compute $\int_{-3}^{1}\int_{2x-1}^{2-x^2} x^2\,dy\,dx$.

b) Sketch the region in $\mathbb{R}^2$ over which the integral $\int_{-3}^{1}\int_{2x-1}^{2-x^2} x^2\,dy\,dx$ is computed on the axes provided.

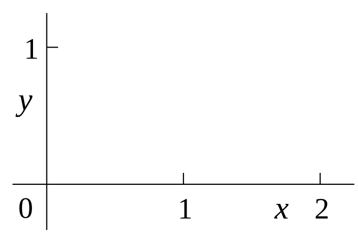
c) Write the integral $\int_{-3}^{1}\int_{2x-1}^{2-x^2} x^2\,dy\,dx$ as a sum of one or more integrals in $dx\,dy$ order. You are *not* asked to compute the result!



(15) 2. In this problem you will compute $\iint_R (x-y^2)^{100}\,dA$ where R is the region in $\mathbb{R}^2$ bounded by $y=0$, $y=1$, $y=\sqrt{x}$, and $y=\sqrt{x-1}$.

a) Sketch the region R on the axes provided.

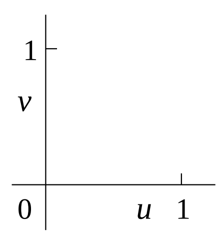
b) Guess a transformation from (u,v) to (x,y) which will greatly simplify the integral.



c) Sketch the region in (u,v) space on the axes provided which corresponds to the region R in (x,y) space.

d) Compute the Jacobian of the transformation from (u,v) to (x,y).

e) Change variables from (x,y) to (u,v) and compute $\iint_R (x-y^2)^{100}\,dA$.

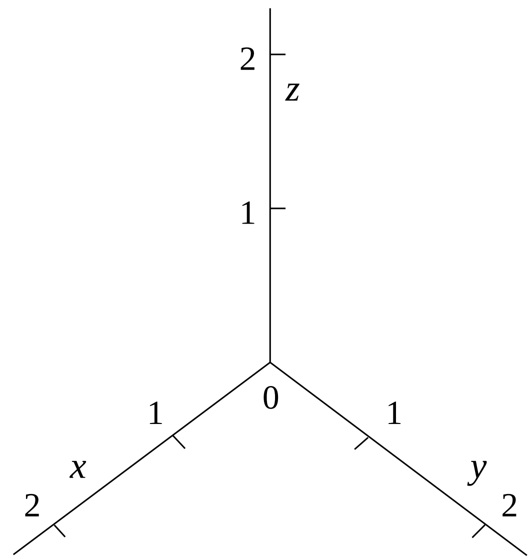


(12) 3. Suppose R is the region in the first octant ($x \geq 0$, $y \geq 0$, and $z \geq 0$) bounded below by the surface $z = x^2 + y$ and is bounded above by the plane $z = 2$.

a) Sketch R on the axes provided.

b) Write the triple integral of xy over R as an iterated integral in $dx\,dy\,dz$ order.

c) Compute $\iiint_R xy\,dx\,dy\,dz$.



(14) 4. a) Suppose D is the unit ball in $\mathbb{R}^3$: those points in $\mathbb{R}^3$ whose distance to the origin is less than or equal to 1. If A is a non-negative real number ($A \geq 0$) compute the triple integral of $(x^2 + y^2 + z^2)^A$ over D. Your answer should depend on A.

Hint The answer for $A = 0$ is well-known!

b) If $A < 0$ the integral of $(x^2+y^2+z^2)^A$ over D is officially an improper integral. Suppose $0 < s < 1$ and D_s is those points in $\mathbb{R}^3$ whose distance to the origin is between s and 1. Compute the triple integral of $(x^2 + y^2 + z^2)^A$ over D_s. Your answer should depend on both s and A. For which A's does the result approach a finite limit as $s \to 0^+$, and what is the limit?

(12) 5. Suppose $f(x, y, z) = Ay + Bz$, and suppose R is the region in $\mathbb{R}^3$ which is contained in the half-space $y \geq 0$ and is bounded above by $z = 1 - x^2 - y^2$ and below by $z = 0$. Find rational numbers A and B so that $\iiint_R f(x, y, z)\,dV = 1 + \pi$.
Comment Rational numbers are quotients of integers. π is not a rational number. Be careful of the r's!

(16) 6. a) Find and classify (local max, local min, or neither) all critical points of $f(x, y) = (x + y)e^{2y-x^2}$.

b) Find and classify (local max, local min, or neither) all critical points of $g(x, y) = (x^2 + y^2 - 1)^{456}$.

Hint Compute a little bit and then *think*!

(16) 7. a) Abstract mathematical theory declares that the function $f(x,y) = x(y+1)$ must attain a maximum value and a minimum value on the set of points in $\mathbb{R}^2$ satisfying $g(x,y) = x^2 + y^2 = 1$. Find the maximum and minimum values using Lagrange multipliers.

b) Abstract mathematical theory declares that the function $f(x,y,s,t) = xy^2s^3t^4$ must attain a maximum value and a minimum value on the set of points in $\mathbb{R}^4$ satisfying $g(x,y,s,t) = x^2+y^2+s^2+t^2 = 1$. Find the maximum and minimum values using Lagrange multipliers.

Second Exam for Math 291, section 1

November 21, 2006

NAME ______________________________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes may be used on this exam other than the formula sheet to be distributed.

Problem Number	Possible Points	Points Earned:
1	15	
2	15	
3	12	
4	14	
5	12	
6	16	
7	16	
Total Points Earned:		