(20) 1. Find all critical points of each function. Describe (as well as you can) the type of each critical point. Explain your conclusions.

a) $f(x,y) = \left(x^2 + y^2\right) e^{x-y}$

b) $g(x,y) = \left(y - x^2\right)^{600}$

(20) 2. Suppose that $F(x,y,z) = y^2 e^{(6x-yz)}$. Note that $F(1,2,3) = 4$.

a) Find a unit vector in the direction is the largest directional derivative of F at $(1,2,3)$. Find the value of that directional derivative.

b) Find an equation for the plane tangent to the surface $y^2 e^{(6x-yz)} = 4$ at the point $(1,2,3)$.

c) Find parametric equations for a line normal to the surface $y^2 e^{(6x-yz)} = 4$ at the point $(1,2,3)$.

(20) 3. The point $p = (-2,1,1)$ satisfies the equation $z^3 + xy^2 z + 1 = 0$. Suppose near the point p that z is defined implicitly by the equation as a differentiable function of x and y.

a) If x is changed from -2 to -2.03 and y is changed from 1 to 1.04, use linear approximation to describe the approximate change in z.

b) What is the value of $\dfrac{\partial^2 z}{\partial x^2}$ at p?

(20) 4. Prove Green's Theorem for the region in the plane bounded by the x-axis and the curve $y = 1 - x^2$ by explicitly computing both sides of the equality for a "general" $P(x,y)\,dx + Q(x,y)\,dy$ (be sure to state what conditions on P and Q are needed) and checking that the two sides are indeed the same.

(20) 5. Suppose a vector field is defined by $\mathbf{F} = \left(y^2 z\right)\mathbf{i} + (2xyz)\,\mathbf{j} + \left(xy^2 + 4z\right)\mathbf{k}$.

a) Determine whether there is a scalar function $P(x,y,z)$ defined everywhere in space such that $\nabla P = \mathbf{F}$. If there is such a P, find it; if there is not, explain why not.

b) Compute the integral $\int_W \mathbf{F} \cdot \mathbf{T}\,ds$, where W is the circular helix whose position vector is given by $\mathbf{R}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}$ for $0 \le t \le 2\pi$. Use information gotten from your answer to a) to help if you wish.

(20) 6. The average value of a function f defined in a region R of $\mathbb{R}^3$ is $\dfrac{\iiint_R f\,dV}{\iiint_R 1\,dV}$. Compute the average distance to the center of a sphere of radius a.

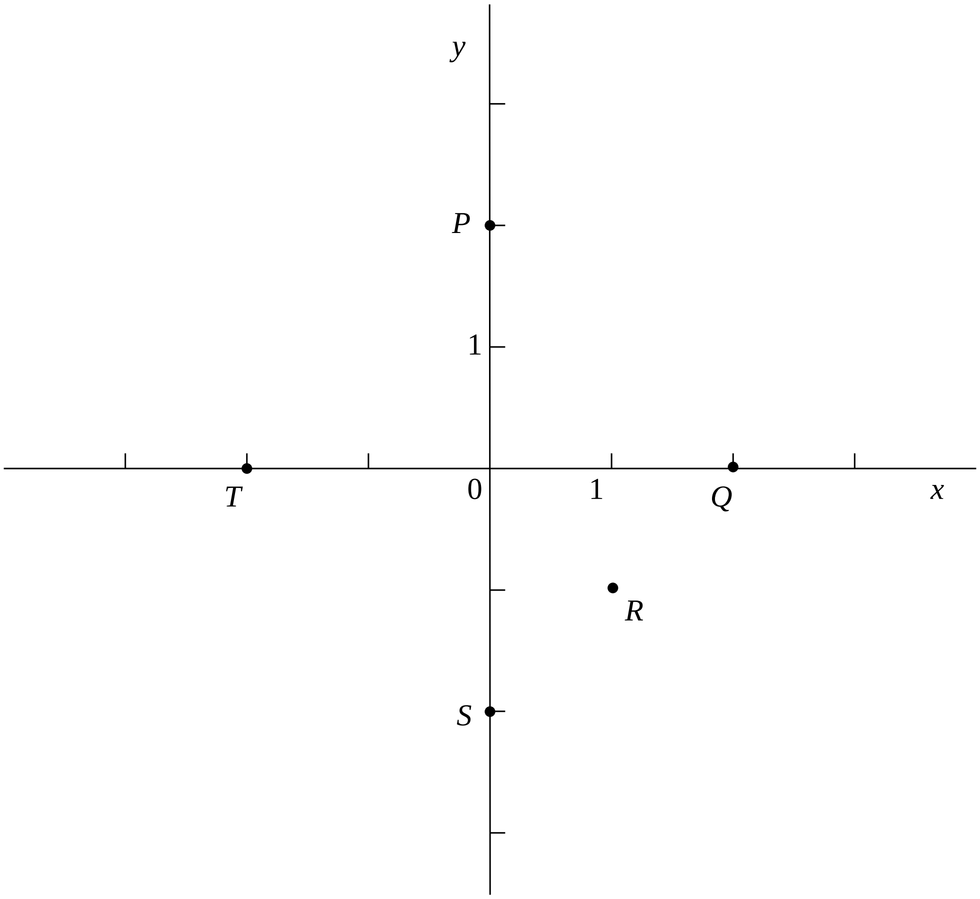
(20) 7. Suppose $f(x,y,z) = xy^2 z^3$.

a) Compute $\displaystyle\int_0^1 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx.$

b) Write the integral in a) as a sum of one or more iterated integrals in $dx\,dy\,dz$ order. You are *not* asked to integrate your answer, only to set it up.

(20) 8. Sketch the three level curves of the function $W(x, y) = ye^x$ which pass through the points $P = (0, 2)$ and $Q = (2, 0)$ and $R = (1, -1)$. **Label each curve** with the **appropriate function value**. Be sure that your drawing is clear and unambiguous.

Also, sketch on the same axes the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S = (0, -2)$ and the point $T = (-2, 0)$.



(20) 9. Suppose $\mathbf{F} = -2xz\mathbf{i} + y^2\mathbf{k}$. **Note** There is *no* $\mathbf{j}$ component in $\mathbf{F}$.

a) Compute curl $\mathbf{F}$.

b) Compute the outward unit normal $\mathbf{n}$ for the sphere $x^2 + y^2 + z^2 = a^2$.

c) If R is any region on the sphere $x^2 + y^2 + z^2 = a^2$, verify that $\iint_R (\text{curl } \mathbf{F}) \cdot \mathbf{n}\, dS = 0$.

d) Suppose C is a simple closed curve on the sphere $x^2 + y^2 + z^2 = a^2$. Show that the value of the line integral $\int_C -2xz\, dx + y^2\, dz$ is 0.

Comment Please *don't* attempt a direct computation! Use c) and one of the big theorems.

(20) 10. a) Verify that the improper integral $\displaystyle\int_0^1 x^{-3/2}\, dx$ does *not* converge.

b) Suppose R is the (roughly) triangular-shaped region in $\mathbb{R}^2$ defined by $y = x^2$, $y = 0$, and $x = 1$. For which values of a and b does the integral $\displaystyle\iint_R x^a y^b\, dA$ converge?

Very difficult

Final Exam for Math 291, section 1

December 22, 2006

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes or calculators may be used on this exam.
A page with formulas will be supplied.

Problem Number	Possible Points	Points Earned:
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total Points Earned:		