`The only acceptable values for` N `in the following paragraph are 2 or 3.` *Please write solutions to $N+1$ of these problems by Wednesday, September 27. These written solutions should be accompanied by explanations using complete English sentences. In this case, all students must work in groups: a group is N students. All students in each group should read all of the group's answers before the work is handed in. Thus all students in each group will therefore be responsible for all answers handed in by the group.*

1. Suppose $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^3$, and the following is known:

$$5 \le \|\mathbf{v}\| \le 10, \quad 30 \le \|\mathbf{w}\| \le 40, \quad 30^\circ \le \theta \le 60^\circ$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$. Then find appropriate over– and under–estimates for the following real numbers using all of the information given:

$$\mathbf{v} \cdot \mathbf{w}, \quad \|\mathbf{v} \times \mathbf{w}\|, \quad \|\mathbf{v} + \mathbf{w}\|.$$

2. Suppose $\mathbf{a}$ and $\mathbf{b}$ are vectors in $\mathbb{R}^3$, which *you* will choose, perhaps differently for each section of this problem. Consider the sequence of vectors $\{\mathbf{A}_n\}$ defined recursively by the following conditions:

$$\mathbf{A}_0 = \mathbf{a}; \qquad \mathbf{A}_{n+1} = \mathbf{A}_n \times \mathbf{b} \;\text{ for } n > 0$$

Thus $\mathbf{A}_3 = \big((\mathbf{a} \times \mathbf{b}) \times \mathbf{b}\big) \times \mathbf{b}$.

a) Can you choose $\mathbf{a}$ and $\mathbf{b}$ so that all of the sequence $\{\mathbf{A}_n\}$ eventually lies in one plane in $\mathbb{R}^3$? If your answer is "yes", give an example and verify your statement. If your answer is "no", explain why. (Experiment. Interesting examples are better than simple examples.)

b) Can you choose $\mathbf{a}$ and $\mathbf{b}$ so that the sequence $\{\mathbf{A}_n\}$ has a limit in $\mathbb{R}^3$? If your answer is "yes", give an example and verify your statement. If your answer is "no", explain why. (Try some experiments. Interesting examples are better than simple examples.)

c) Can you choose $\mathbf{a}$ and $\mathbf{b}$ so that the sequence $\{\mathbf{A}_n\}$ is eventually periodic? If your answer is "yes", give an example and verify your statement. If your answer is "no", explain why. (Again, interesting examples are better than simple examples.)

3. Find equations for two orthogonal planes both of which contain the line $\mathbf{v} = (1,0,3) + t(-1,2,1)$, one of which passes through the origin.

4. Find the distance between the pair of skew lines given below ("skew" in this case means a bit more than non-intersecting – it means that they are non-intersecting *and* not parallel):

$$\text{The line } L_1 \text{ is } \begin{cases} x = 2t+1 \\ y = -t-1 \\ z = 3t \end{cases}. \quad \text{The line } L_2 \text{ is } \begin{cases} x = 3s+2 \\ y = 5s-2 \\ z = -4 \end{cases}.$$

OVER

Comment I used different letters (t and s) for the parameters in the two lines. This was to help, because these letters are "dummy" variables (similar logically to the letters in a definite integral: surely $\int_0^1 w^2 dw$ and $\int_0^1 u^2 du$ are the same).

5. Is the point (1,2,3) on a tangent line of the twisted cubic $\mathbf{c}(t) = (t, t^2, t^3)$?

6. A function $c : \mathbb{R} \to \mathbb{R}^2$ is called *smooth* if c is differentiable. Physically such functions should represent motion that has no jerks or kinks.

a) Suppose $q : \mathbb{R} \to \mathbb{R}$ is defined by $q(x) = \begin{cases} x^{100} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$. Sketch $y = q(x)$ and prove that q is differentiable. (Do this by first computing the derivative algorithmically for $x < 0$ and $x > 0$ – this is easy. Compute $q'(0)$ by looking directly at the definition and computing the limit from both sides.)

b) Sketch the smooth curve defined by $c(t) = (q(t), q(-t))$.

c) Explain why you've drawn a picture of smooth motion. (Hardest part of the question!)