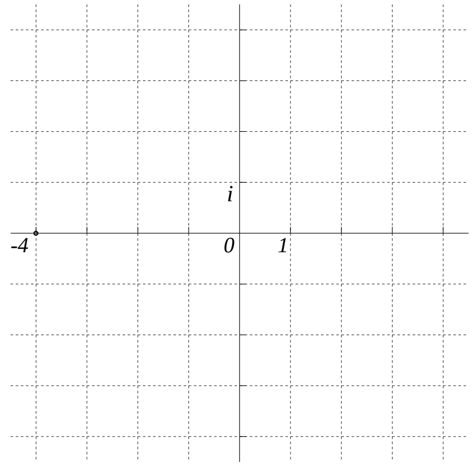
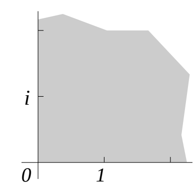
(10) 1. Describe all solutions of $z^4 = -4$ algebraically in rectangular form. Sketch the solutions on the axes provided.

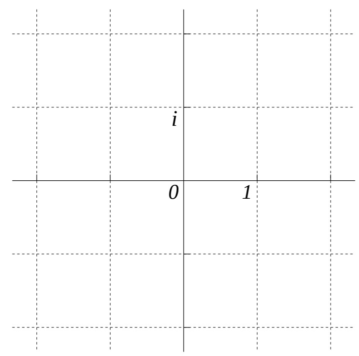


(14) 2. In this problem the open first quadrant of $\mathbb{C}$ will be denoted by Q. A number z is in Q if both $\operatorname{Re} z > 0$ and $\operatorname{Im} z > 0$. A portion of Q is shown to the right.

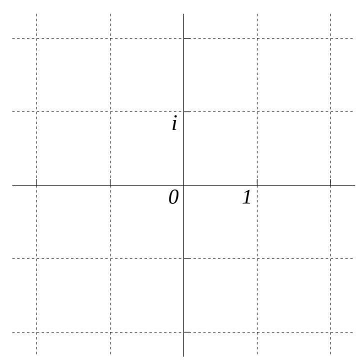


a) Write the complex polar expression for z and indicate what restrictions on r and θ are needed so that z is guaranteed to be in Q.

b) What is the image of Q under the mapping $z \mapsto z^2$? Indicate the portion of the plane which is the image on the axes to the right and briefly explain your drawing algebraically.



c) What is the image of Q under the mapping $z \mapsto \dfrac{i}{z}$? Indicate the portion of the plane which is the image on the axes to the right and briefly explain your drawing algebraically.



d) Verify the following assertion: if z is in Q, then $\dfrac{z^3 + i}{z}$ is in the open upper half plane (those complex numbers whose imaginary part is positive). You may use the conclusions of parts b) and a) combined with geometric reasoning, or some other method.

(12) 3. Suppose that $f(z)$ is an analytic function with real part $u(x, y)$ and imaginary part $v(x, y)$, and that $u(x, y) = \big(v(x, y)\big)^2$ always. Show that $f(z)$ must be constant.

Hint The Cauchy-Riemann equations.

(10) 4. Find the values of A so that

$$e^{3x}\sin(Ay)$$

is harmonic. Find a harmonic conjugate for each of these harmonic functions.

(12) 5. a) Find all values of $\log(-1)$ and $(-1)^i$.

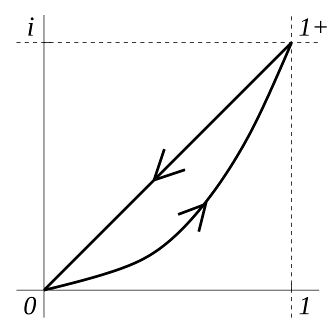
b) Provide an explicit pair of complex numbers z and w so that

$$\mathrm{Log}(zw) \neq \mathrm{Log}\, z + \mathrm{Log}\, w.$$

(10) 6. Define $\sin z$ and $\cos z$ in some way appropriate for this course, and use your definitions to prove that $(\sin z)^2 + (\cos z)^2 = 1$ for all complex z.

(12) 7. Compute $\int_C x^2\, dz$ where C is the simple closed curve obtained by following the parabola $y = x^2$ from 0 to $1+i$, and then following the line $y = x$ from $1+i$ to 0.

The curve is displayed to the right.

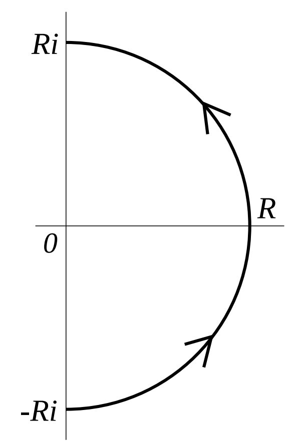


(12) 8. Verify that

$$\lim_{R\to\infty} \int_{C_R} \frac{e^{-z} + 2z}{z^3 + 6z^2}\, dz = 0$$

where C_R is the semicircle of radius R and center 0 in the right half plane going from $-Ri$ to Ri as shown.

Show details of your estimates.



(8) 9. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n(n+1)}{(3+4i)^n} z^n.$$

First Exam for Math 403, section 2

March 3, 2005

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes, texts, or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	10	
2	14	
3	12	
4	10	
5	12	
6	10	
7	12	
8	12	
9	8	
Total Points Earned:		