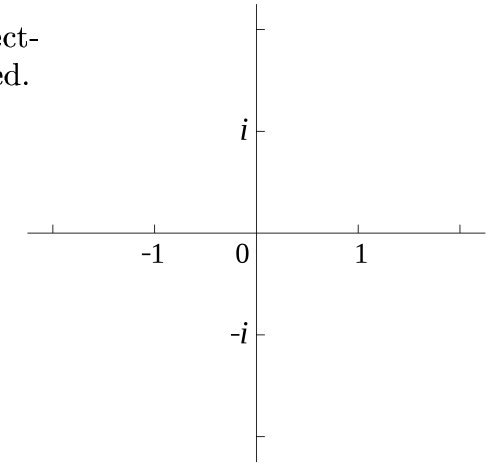
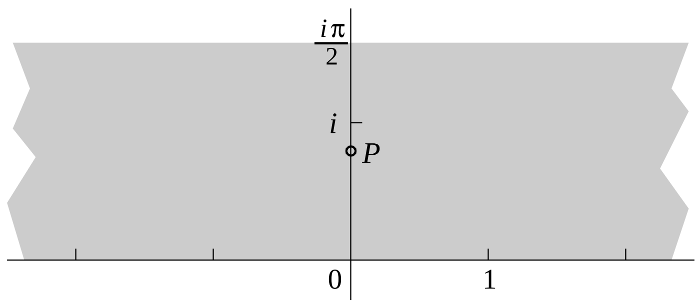
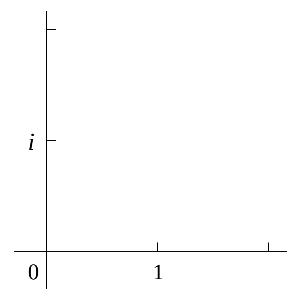
(9) 1. Describe all solutions of $z^3 = -8i$ algebraically in rectangular form. Sketch the solutions on the axes provided.



(14) 2. In this problem U is the open horizontal strip defined by $0 < \operatorname{Im} z < \frac{\pi}{2}$ and P is the point $\frac{i\pi}{4}$ in U. Part of U is shown below, and P is also indicated.



a) Suppose V is the image of U under the exponential mapping. Sketch V on the axes provided to the right. Write an algebraic description of V below. Also label the point Q, the image of P under the exponential mapping, on your sketch, and write rectangular complex coordinates for Q.



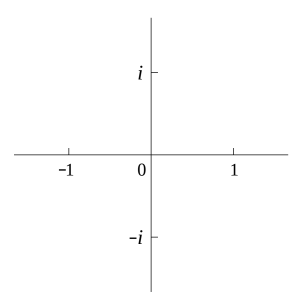
Algebraic description of V (Complex polar or rectangular)

Complex rectangular coordinates for Q

b) Suppose that W is the image of V (NOTE: V) under the cubing mapping (that is, $z \to z^3$). Sketch W on the axes provided below. Write an algebraic description of W below. Additionally, label the point R, the image of Q (NOTE: Q) under the cubing mapping, on your sketch, and write rectangular complex coordinates for R.

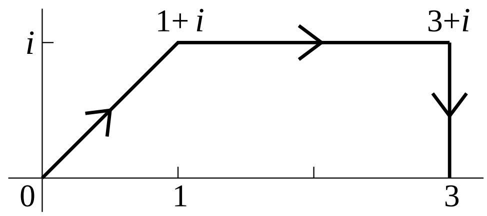
Algebraic description of W (Complex polar or rectangular)

Complex rectangular coordinates for R



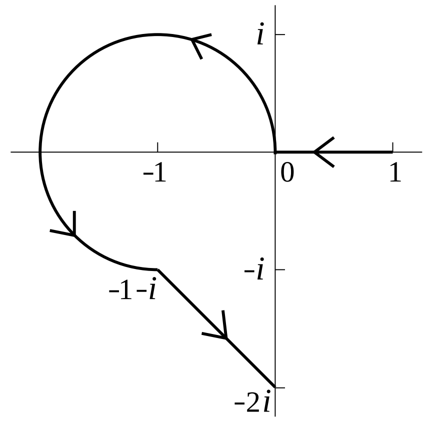
(12) 3. a) Find all values in rectangular complex form of $(1+i)^i$ and $(1+i)^2$.
b) Provide an explicit pair of complex numbers z and w so that $\mathrm{Log}(zw) \neq \mathrm{Log}\, z + \mathrm{Log}\, w$.

(13) 4. a) Compute $\int_C x\, dz$ where C is the curve given by a line segment from 0 to $1+i$ followed by a line segment from $1+i$ to $3+i$, and then followed by a line segment from $3+i$ to 3. This curve is displayed to the right.



Note You do *not* need to "simplify" your answer.

b) Compute $\int_C (z^2+1)\, dz$ where C is the curve given by a line segment from 1 to 0, followed by a circular arc of radius 1 centered at -1, connected 0 to $-1-i$, followed by a line segment from $-1-i$ to $-2i$. This curve is displayed to the right.



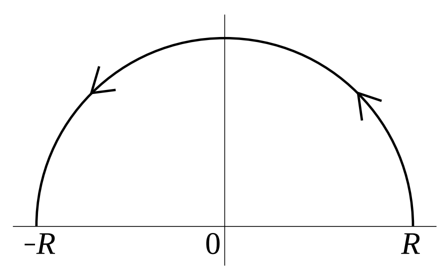
Note You do *not* need to "simplify" your answer.

(16) 5. a) Suppose that $h(x,y)$ is a harmonic function. Verify that $\frac{\partial h}{\partial x} - i\frac{\partial h}{\partial y}$ is analytic.
b) Verify that $3x^2y - y^3 + y$ is harmonic, and find all harmonic conjugates.

(12) 6. a) Define $\sin(z)$ and $\cos(z)$ in some way appropriate for this course, and use your definitions to prove that $\sin(2z) = 2\sin(z)\cos(z)$ for all complex z.

b) Find a value of z so that $|\sin z| > 2$. Verify your assertion.

(12) 7. Verify that $\lim_{R\to\infty} \int_{C_R} \frac{e^{iz}-3z}{z^4+100}\, dz = 0$ where C_R is the semicircle of radius R and center 0 in the upper halfplane going from R to $-R$ as shown.



Show details of your estimates. (How large should R be so division by 0 is avoided?)

(12) 8. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \left(\frac{n}{4^n}\right) z^n$ and find a "closed form" (that is, a simple expression) for the sum of this power series. Use this closed form to compute the exact value of the sum $\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$.

First Exam for Math 403, section 1

March 10, 2008

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes, texts, or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	9	
2	14	
3	12	
4	13	
5	16	
6	12	
7	12	
8	12	
Total Points Earned:		