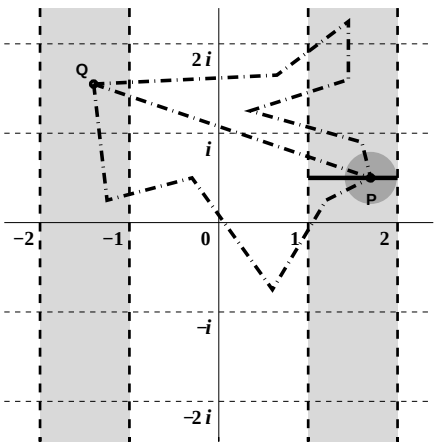
Math 403, section 1 **Answers to the "workshop"** **February 5, 2008**

Two students must work on this together!!!

Well, it did get you to talk to each other, didn't it? Here are some possible answers.

1. Suppose A is the collection of complex numbers, z, which satisfy $1 < |\mathrm{Re}\, z| < 2$. Sketch A as well as possible on the axes to the right. Answer these questions and briefly explain your answers.



a) Is A open? Give some explanation.

Answer Yes, A is open. If **P** is a point in A, draw a horizontal line segment through **P** touching the closest two of the lines $\mathrm{Re}\, z = \pm 1$ or ± 2. **P** divides that segment in two pieces. Take a disc centered at **P** whose radius is the minimum of the lengths of those two pieces. That disc lies in A.

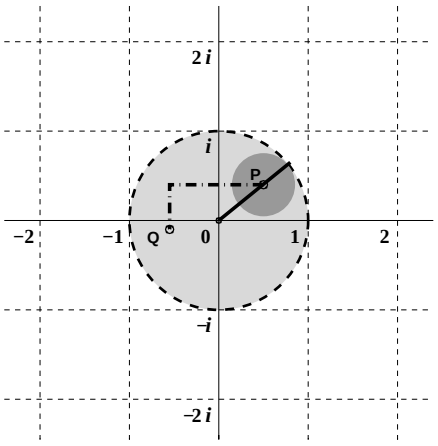
b) Is A connected? Give some explanation.

Answer A is not connected. Take points **P** and **Q** whose real parts have different signs. Any path between them must have a point on it with real part equal to 0 and therefore will not contained in A.

c) What is the boundary of A? Give some explanation.

Answer The boundary of A is the four lines $\mathrm{Re}\, z = \pm 1$ or ± 2. A disc centered around a point on any of these lines, for example, $\mathrm{Re}\, z = 1$, has points in the disc with $\mathrm{Re}\, z > 1$ and with $\mathrm{Re}\, z < 2$, hence has points in A. It also has points with $\mathrm{Re}\, z < 1$ and also $\mathrm{Re}\, z > -1$, so it has points not in A.
Other points in $\mathbb{C}$ and not in A have discs around them entirely disjoint from B. Just take a disc of radius the smallest distance to the four lines mentioned.

2. Suppose B is the collection of complex numbers, z, which satisfy $0 < |z| < 1$. Sketch B as well as possible on the axes to the right. Answer these questions and briefly explain your answers.



a) Is B open? Give some explanation.

Answer Yes, B is open. If **P** is a point in B, draw a radial line segment from 0 through **P** to the unit circle. A disc whose radius is the smaller of the length of the two line segments formed when **P** is taken out of the radial segment will be inside B.

b) Is B connected? Give some explanation.

Answer Yes, B is connected. A path formed by a horizontal and a vertical line segment will connect any two points of B. See the picture where such a path is shown connecting **P** and **Q**.

c) What is the boundary of B? Give some explanation.

Answer The boundary of B consists of all points on the unit circle (radius 1 and center 0) and 0 itself.
Note that 0 is not in B, so any disc centered at 0 with positive radius contains 0 (not in B!) and points at small positive distance from 0, certainly in B.
Now take a point on the unit circle. Any disc centered at such a point will have part of the disc at distance from 0 less than 1 but positive (and therefore in B) and will have part of the disc at distance from 0 more than 1 (and thus not in B). So such points are also on the boundary.
Other points in $\mathbb{C}$ are not on the boundary because they are either inside B which is open or at distance from 0 greater than 1, and therefore have discs around them containing no points of B.