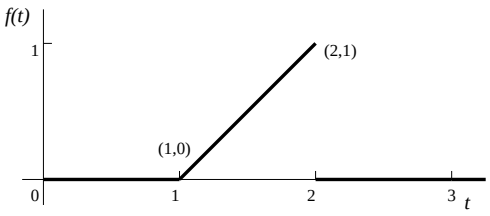
(16) 1. Here is a graph of the function $f(t)$ which is piecewise linear.



a) Use the definition of the Laplace transform to find the Laplace transform $F(s)$ of the function $f(t)$.

b) Certainly $\int_0^\infty f(t)\,dt = \frac{1}{2}$. Use l'Hopital's rule to verify that $\lim_{s\to 0^+} F(s) = \frac{1}{2}$. Be sure to indicate why l'Hopital's rule applies each time you use it.

(14) 2. a) Use Laplace transforms to solve the initial value problem $y''-3y' = 1$ with $\begin{cases} y(0) = 1 \\ y'(0) = -1 \end{cases}$.

b) Check that your answer satisfies the initial conditions.

$y(0) =$ _____ . $y'(t) =$ _____ so that $y'(0) =$ _____ .

(12) 3. Find the Laplace transform of $H(t-3)\left(t^2 + e^{5t} + 1\right)$.

(8) 4. Compute the convolution of e^{5t} and e^{8t}.

(20) 5. a) Solve the initial value problem $y'' + y = H(t-\pi) + \delta(t-\frac{\pi}{2})$ with $\begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases}$.

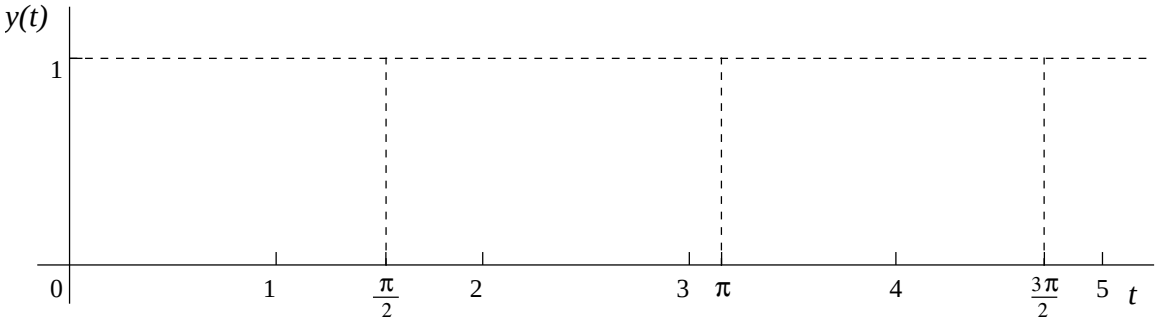
b) Write formulas *without* Heaviside functions for $y(t)$ in the indicated intervals:

If $0 < t < \frac{\pi}{2}$ then $y(t) =$ _____ .

If $\frac{\pi}{2} < t < \pi$ then $y(t) =$ _____ .

If $\pi < t$ then $y(t) =$ _____ .

c) Graph $y(t)$ as well as you can on the axes below.



d) For which t in the interval $0 < t < 5$ is $y(t)$ differentiable?

ANSWER: _____

(14) 6. a) Find the Laplace transform of this linear first-order system of ordinary differential equations $\begin{cases} 2x'(t) + 3x(t) + y'(t) = e^t \\ x'(t) + x(t) - y'(t) + 5y(t) = \sin t \end{cases}$ with initial conditions $\begin{cases} x(0) = 1 \\ y(0) = 0 \end{cases}$.

b) Find an expression for the Laplace transform, $X(s)$, of $x(t)$ which does not involve the Laplace transforms of $y(t)$. Do **not** simplify your answer! Do **not** try to compute $x(t)$!

(16) 7. Show that $\mathbf{u} = (1, -1, 0, 1, 1)$ and $\mathbf{v} = (2, 2, -2, 2, 2)$ and $\mathbf{w} = (1, 5, -3, 1, 1)$ are linearly dependent in $\mathbb{R}^5$.

First Exam for Math 421, section 2

February 24, 2004

NAME _____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	16	
2	14	
3	12	
4	8	
5	20	
6	14	
7	16	
Total Points Earned:		