

Math 421

Some Fourier series examples

Here is a specification of $f(x)$:

```
f:=x->piecewise(x>0,1,0);
```

These Maple commands compute the Fourier coefficients of $f(x)$:

```
q:=n->(1/Pi)*int(f(x)*cos(n*x),x=-Pi..Pi);
```

```
r:=n->(1/Pi)*int(f(x)*sin(n*x),x=-Pi..Pi);
```

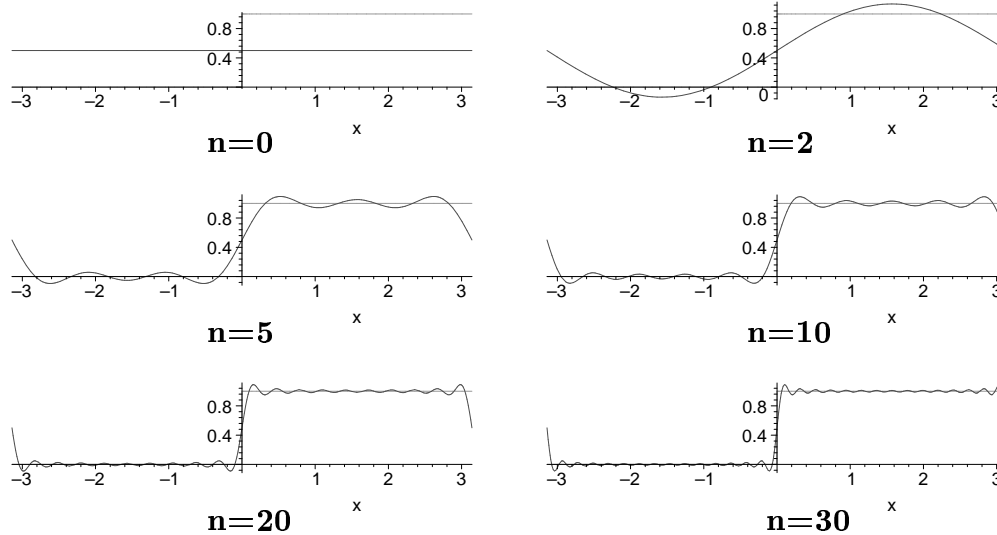
This command computes a partial sum of the Fourier series of $f(x)$ and groups it together with $f(x)$:

```
Q:=k->{f(x),(1/2)*q(0)+sum(q(n)*cos(n*x)+r(n)*sin(n*x),n=1..k)}
```

This command plots the partial sum and the original function:

```
QQ:=n->plot(Q(n),x=-Pi..Pi,scaling=constrained);
```

Now some pictures:



Here's an effort to approximate the Parseval equality. This first command computes a partial sum of the infinite series corresponding to one side of the equality:

```
s:=k->(1/2)*q(0)+sum(q(n)*cos(n*x)+r(n)*sin(n*x),n=1..k)
```

The next command computes this sum symbolically if possible, then computes it numerically, then computes the integral symbolically if possible, then computes it numerically:

```
pars:=k->[s(k),evalf(s(k)),
```

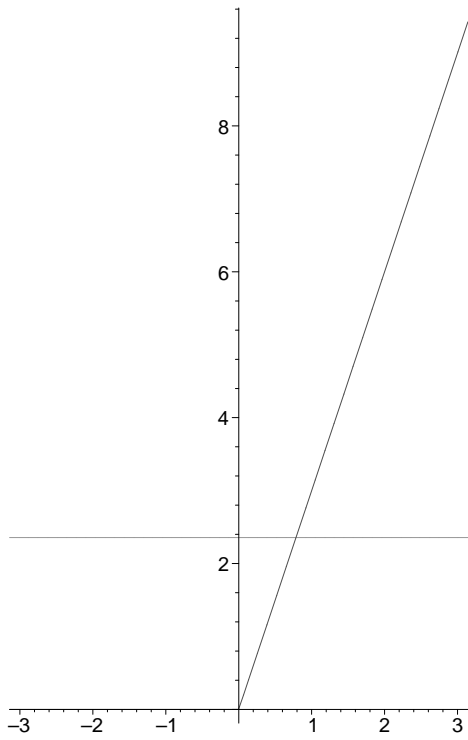
```
(1/Pi)*int(f(x)^2,x=-Pi..Pi),evalf((1/Pi)*int(f(x)^2,x=-Pi..Pi))];
```

Here is the result of `pars(10)` and then `pars(10)`:

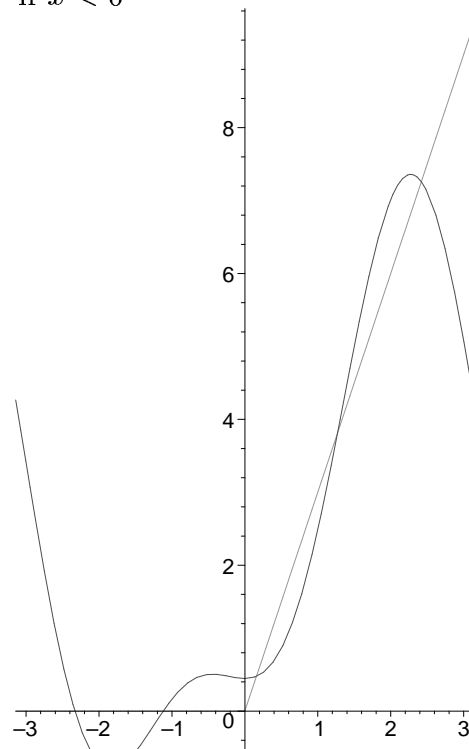
```
[1/2+469876/99225Pi^2, .9798023932, 1, 1.]
```

```
[1/2+204698374253288/42337793743245Pi^2, .9898762955, 1, 1.]
```

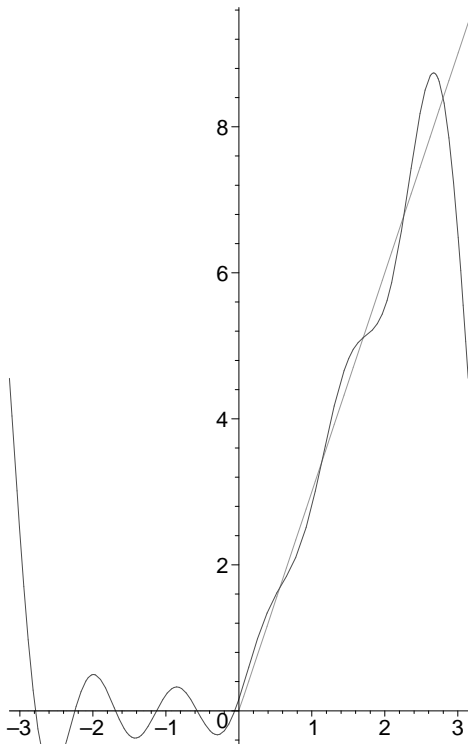
Now here's the function $f(x) = \begin{cases} 3x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$, and some pictures:



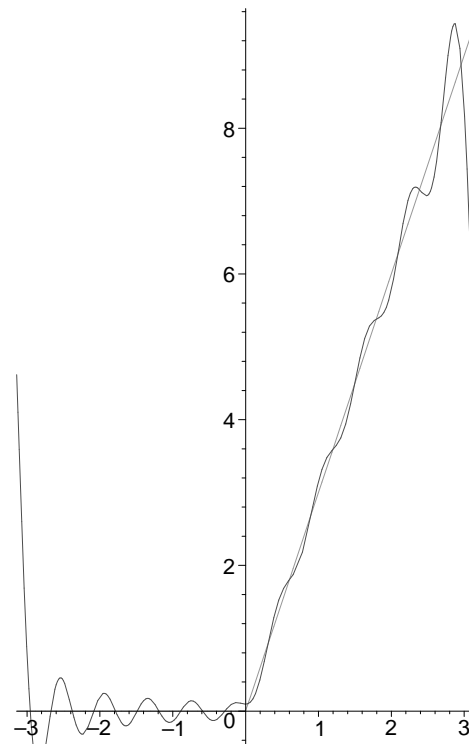
n=0



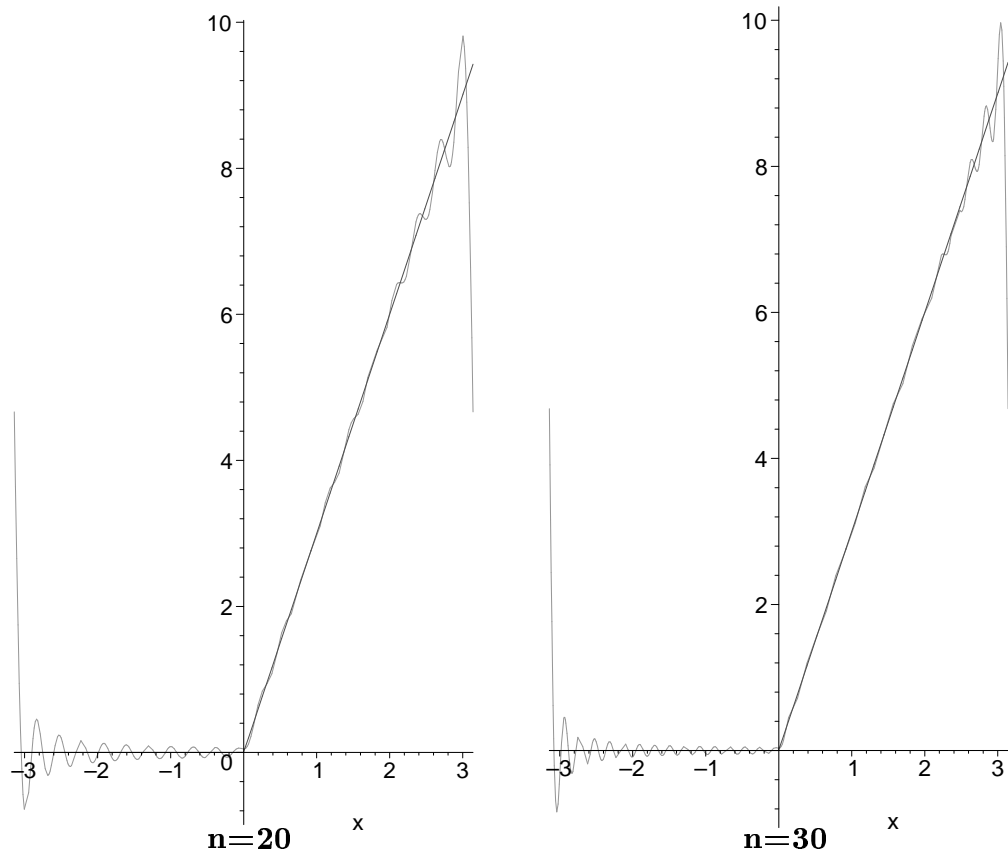
n=2



n=5



n=10

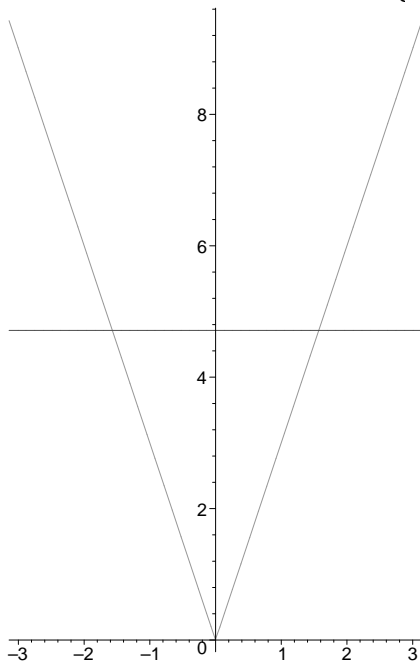


Parseval, $n = 10$:

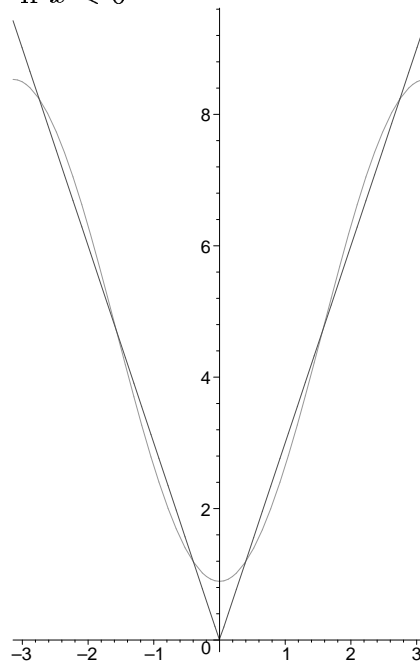
$$\left[\frac{9 \pi^2}{8} + \frac{1968329}{141120} + \frac{39954021844}{1093955625 \pi^2}, 28.75172001, 3 \pi^2, 29.60881321 \right]$$

Maple also tells me that the 3rd cosine Fourier coefficient is $-\frac{2}{3\pi}$ and the 5th sine Fourier coefficient is $\frac{3}{5}$. Let's check this, and see if we can get an interesting expression for Parseval's equality.

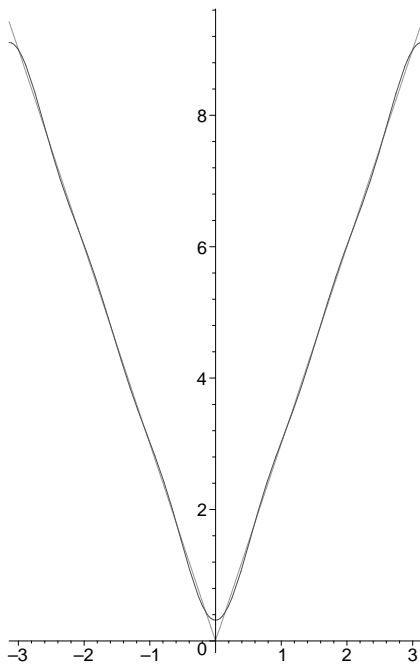
Here's the function $f(x) = \begin{cases} 3x & \text{if } x \geq 0 \\ -3x & \text{if } x < 0 \end{cases}$, the *even extension* of the original $f(x)$.



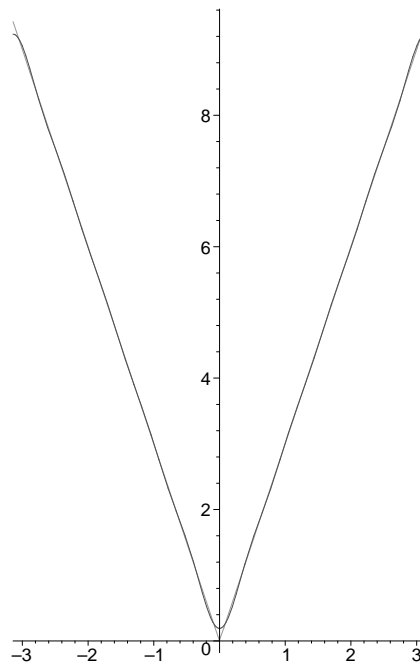
n=0



n=2



n=5

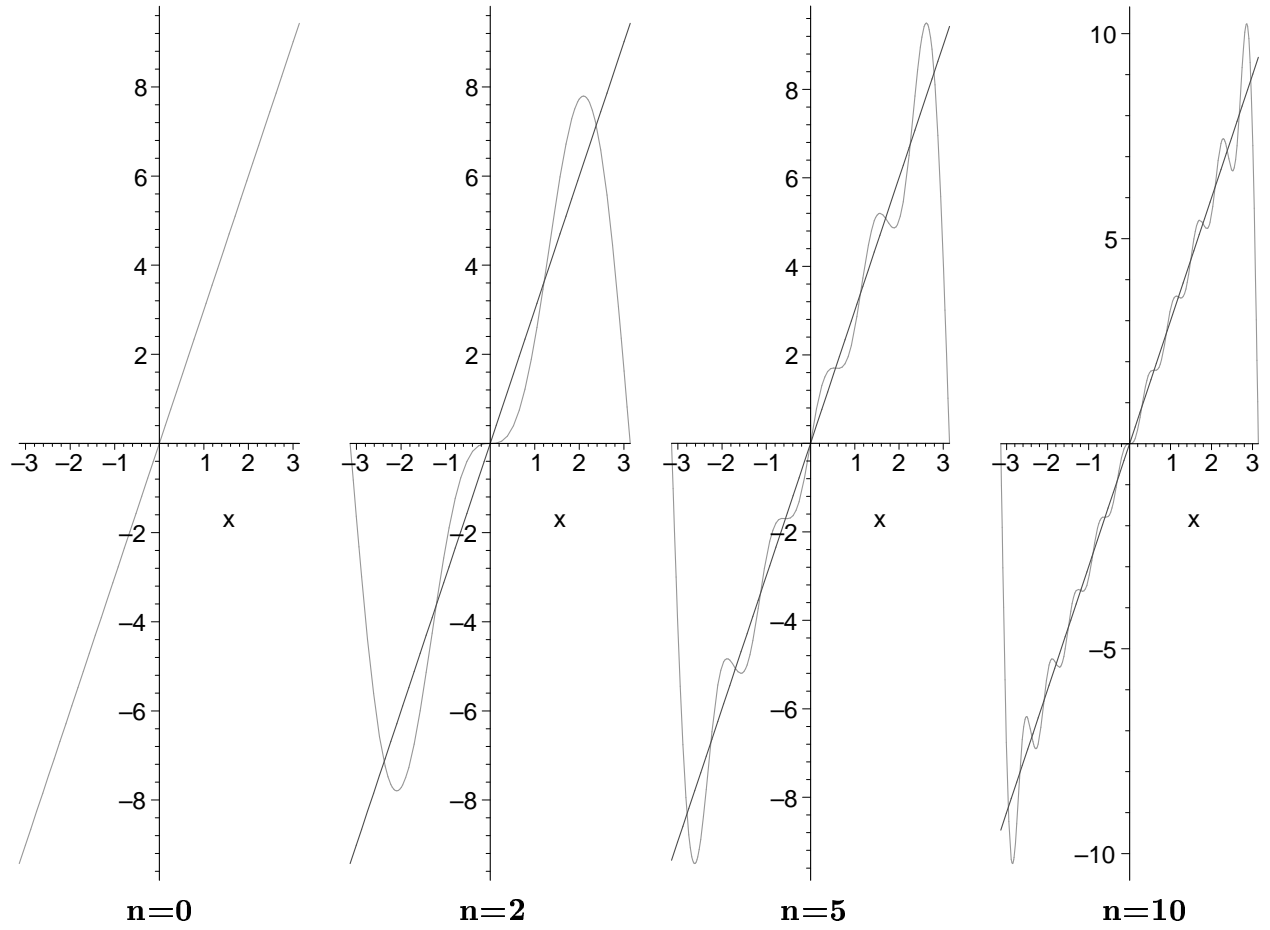


n=10

Parseval, $n = 10$:

$$\frac{9 \pi^2}{2} + \frac{159816087376}{1093955625 \pi^2}, \quad [59.21524174, 6\pi^2, 59.21762642]$$

Here's $f(x) = 3x$ (for all x), the *odd extension* of the original $f(x)$.



Parseval, $n = 10$:

$$\left[\frac{9 \pi^2}{2} + \frac{159816087376}{1093955625\pi^2}, 59.21524174, 6 \pi^2, 59.21762642 \right]$$