2/16/2004

Sample problems for exam #1 in Math 421

The exam will cover up to and including material in the first 9 lectures of the course (Laplace transform; 5.4, 5.5, 6.1). Thus material up to and including the lecture on Tuesday, February 17, will be included.

Here are some sample problems, most from past exams in Math 421. This set of review problems is definitely longer than the exam will be. Here and on the exam, a bare answer will likely not be sufficient to get full credit. Supporting computation or a description of your reasoning must be supplied.

1. Find the inverse Laplace transform of $\dfrac{se^{-s}}{s^2-1} - \dfrac{s^2}{(s+2)^2}$,

2. Compute the convolution $f(t) = (H(t)t^2) * (H(t)\cos(t))$ and its Laplace transform.

3. Use the Laplace transform to solve the initial value problem $y''+2y'-3y = H(t-2)(t-1)$ with $y(0) = 1$ and $y'(0) = -1$.

4. Consider the function $f(t) = \begin{cases} 0 & 0 < t < 1 \\ t & 1 < t < 3 \\ 1 & 3 < t \end{cases}$

a) Sketch $f(t)$.

b) Express $f(t)$ in terms of Heaviside functions.

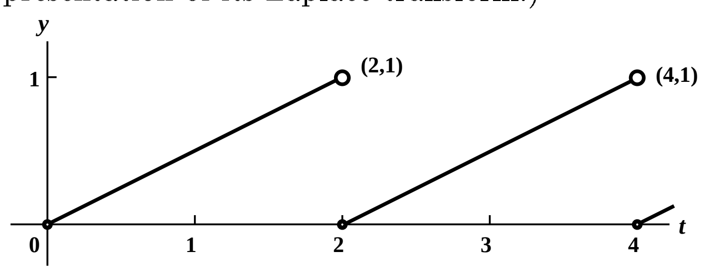
c) Calculate the Laplace transform of $f(t)$.

5. Find the inverse Laplace transform of $\dfrac{s}{(s^2+36)^2}$.

6. Solve for $y(t)$ where $y(t) = 1 - \displaystyle\int_0^t y(\tau)e^t e^{-\tau}\,d\tau$.

7. Evaluate $\displaystyle\int_0^\infty \delta(t-e)\ln(t) + \delta(t-\pi)H(t-2)\,dt$.

8. What is the Laplace transform of the function shown? (The function is piecewise linear, and extends periodically for all positive t with period 2. Try to find a compact representation of its Laplace transform.)



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9. a) Find the Laplace transform of this first-order system of ordinary differential equations:

$$\begin{cases} x(t) & +3x'(t) & +y(t) & +4y'(t) & & +z'(t) & = e^t \\ & & -2y(t) & +y'(t) & -z(t) & +3z'(t) & = t^2 \\ & & y(t) & +3y'(t) & & +4z'(t) & = 2 \end{cases}$$

with the initial conditions $x(0) = 1$, $y(0) = 2$, and $z(0) = 0$.

b) Find an expression for the Laplace transform $Y(s)$ of $y(t)$ which does not involve the Laplace transforms of the other unknown functions. You are not asked to compute $y(t)$ from this information.

10. Give an example of a function defined on $[0, \infty)$ which does *not* have a Laplace transform (that is, make it grow too fast).

11. A student asserts that the Laplace transform of a function is always decreasing. This is incorrect. Can you give an explicit example of a function whose Laplace transform at 10 is larger than its value at 5?

12. Are the vectors $(0, 1, 3, 4)$, $(1, 2, 4, -1)$, and $(-2, 1, 1, 0)$ linearly independent in $\mathbb{R}^4$?

13. Is $(1, 1, 1, 1, 1)$ in the span of the vectors $(0, 2, 0, 2, 1)$, $(1, 0, 1, 0, 1)$ and $(2, 2, 2, 2, 2)$ in $\mathbb{R}^5$?

14. Give an example of a subspace of $\mathbb{R}^7$ of dimension 4. What is a basis of your example?

15. If $A = \begin{pmatrix} 3 & 0 & -1 \\ 1 & 2 & 2 \\ 4 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 2 & 0 \\ 2 & -1 & 1 \end{pmatrix}$ compute any of the matrices following which are defined, and explain why the others are *not* defined.

$$A^2 \qquad AB \qquad BA \qquad B^2 \qquad A^2 + 3A \qquad A + B$$

16. Use the definition of Laplace transform to find the Laplace transform of t.