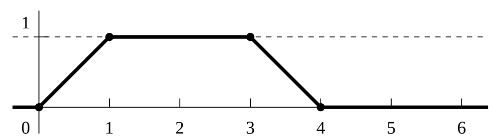
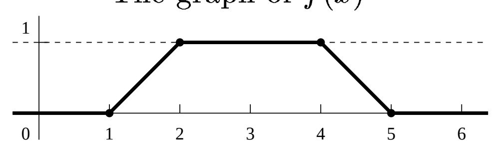
(12) 1. Complete the definitions.
a) Suppose v_1, v_2, … and v_t are vectors in $\mathbb{R}^n$. Then v_1, v_2, … and v_t are *linearly independent* if
b) Suppose A is an n by n matrix. λ is an *eigenvalue* of A if

(22) 2. Suppose that $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. **Note** A is *not* symmetric!
a) Compute the characteristic polynomial of A.
b) Find the eigenvalues of A.
c) Find a basis of $\mathbb{R}^3$ consisting of eigenvectors of A.
d) Find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$.
e) Find P^{-1}.
f) Compute $Z = AP$.
g) Compute $P^{-1}Z$ using the results of d) and e).
h) Write A as a product of D and P and P^{-1} (in the correct order!) and use this information to compute A^6. **Note** The entries in the answer are 0, 1, 31, or 32.

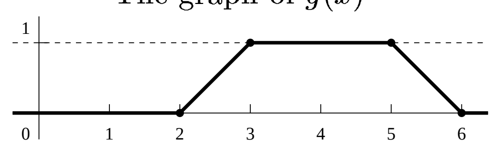
(16) 3.In this problem the functions $f(x)$ and $g(x)$ and $h(x)$ are piecewise linear functions. Parts of their graphs are shown to the right. The domains of these functions are all real numbers (all of $\mathbb{R}$). The functions are 0 where the graphs are not shown.
a) Prove that the functions $f(x)$ and $g(x)$ and $h(x)$ are linearly independent.



The graph of $f(x)$

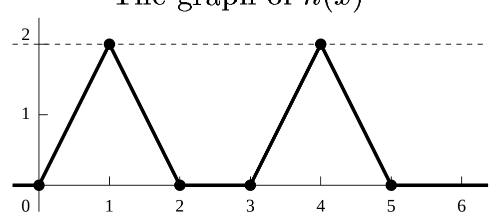


The graph of $g(x)$



The graph of $h(x)$

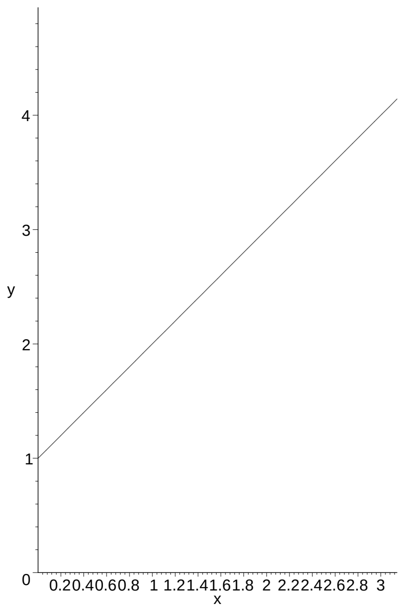
b) The function $Q(x)$ is piecewise linear and part of its graph is shown to the right. The domain of $Q(x)$ is all real numbers (all of $\mathbb{R}$) and the function $Q(x)$ is 0 where the graph is not shown. Can $Q(x)$ be written as a linear combination of $f(x)$ and $g(x)$ and $h(x)$?



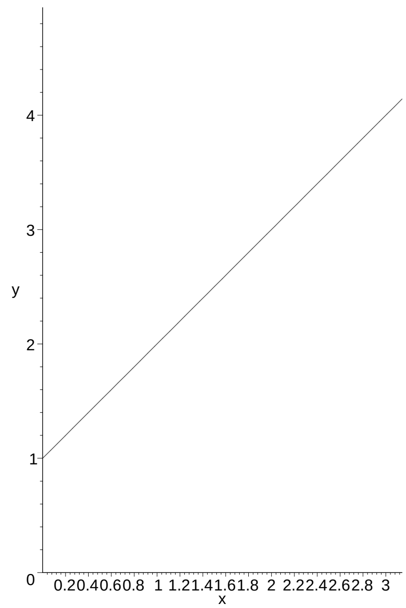
The graph of $Q(x)$

(12) 4. Suppose M is the matrix $\begin{pmatrix} a+b & 0 & 0 & 1 \\ b & -1 & 0 & 1 \\ c & 2 & -1 & 0 \\ b-c & 1 & -1 & 0 \end{pmatrix}$. Prove that M is <u>not invertible</u> exactly when the vector (a, b, c) in $\mathbb{R}^3$ is perpendicular to the vector $(1, 1, -2)$ in $\mathbb{R}^3$.

(18) 5. In this problem, $f(x) = x + 1$. a) Compute $\int f(x)\sin(nx)\,dx$.
Comment Yes, this is an indefinite integral. Yes, you should integrate by parts. Yes, you can *guess* the answer, but then you must verify the answer by differentiation.
b) Compute $b_n = \int_0^\pi f(x)\sin(nx)\,dx$ as explicitly as you can when n is a positive integer.
c) Give exact values for b_1 and b_2 and b_3 and b_4.
d) Suppose $g(x) = \frac{2}{\pi}\sum_{n=1}^{100} b_n \sin(nx)$ and $h(x) = \frac{2}{\pi}\sum_{n=1}^{\infty} b_n \sin(nx)$. Below are two graphs of $f(x) = x + 1$ for x in $[0, \pi]$.
Sketch a reasonable approximation to $g(x)$ on the left graph.
Sketch a reasonable approximation to $h(x)$ on the right graph.



Graph of $g(x)$, the 100^{th} partial sum of the Fourier sine series on $[0, \pi]$



Graph of $h(x)$, the sum of the whole Fourier sine series on $[0, \pi]$

(12) 6. Suppose $f(x) = 3\sin(2x) - 5\cos(3x) + 2\cos(4x)$. Which of these integrals is larger:

$$\int_{-\pi}^{\pi} \left(f(x)\right)^2 dx \text{ or } \int_{-\pi}^{\pi} \left(f'(x)\right)^2 dx?$$

The following problem statement as presented on the exam was not well-written. Better alternatives are written below.

(8) 7. Suppose $f(x) = x + x^4$ for x in $[0, \pi]$.
a) If $F(x)$ is the odd extension of $f(x)$ to $[-\pi, \pi]$, write a formula or formulas for $F(x)$. Be sure you specify $F(x)$ for all x's in $[-\pi, \pi]$. Which terms *must* be 0 in the Fourier series of $F(x)$?
Alternative statement Suppose $F(x)$ is the odd extension of $f(x)$ to $[-\pi, \pi]$. Write a polynomial formula for $F(x)$ when $x < 0$. Which Fourier coefficients of $F(x)$ must be 0?
b) If $G(x)$ is the even extension of $f(x)$ to $[-\pi, \pi]$, write a formula or formulas for $G(x)$. Be sure you specify $G(x)$ for all x's in $[-\pi, \pi]$. Which terms *must* be 0 in the Fourier series of $G(x)$?
Alternative statement Suppose $G(x)$ is the even extension of $f(x)$ to $[-\pi, \pi]$. Write a polynomial formula for $G(x)$ when $x < 0$. Which Fourier coefficients of $G(x)$ must be 0?

Second Exam for Math 421, section 3

November 18, 2004

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	22	
3	16	
4	12	
5	18	
6	12	
7	8	
Total Points Earned:		