is not invertible. 2) With this value of x compute the eigenvalues of M.

4   (Worth 25 pts) 1) Compute the eigenvalues of the matrix
$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}.$$

2) Compute an eigenvector of A for each eigenvalue of A. 3) Find an orthogonal 3×3 matrix Γ such that the matrix $\Gamma^{-1} A \Gamma$ is diagonal. Write down $\Gamma^{-1} A \Gamma$.

5   (Worth 10 Pts) Using the orthonormality relations for the system of functions $\frac{1}{\sqrt{\pi}} \cos mx$, $\frac{1}{\sqrt{\pi}} \sin nx$, compute the integral

$$\frac{1}{\pi} \int_{-\pi}^{\pi} \left(3 \cos 5x + 4 \sin 4x + 5 \cos 3x \right)^2 dx$$

6   (Worth 25 pts) Compute the Fourier series (coefficients: a_n and b_n) of the periodic function with period 2π, $g(x) = 1 - \frac{1}{\pi}|x|$ for $-\pi \leq x \leq \pi$. Is this Fourier series equal to $g(x)$ for every x? Compute the sum of the series

$$\sigma = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} \left(a_n^2 + b_n^2 \right).$$

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