a) $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + 2\beta \frac{\partial u}{\partial t}$ $0<\beta<1$ $t>0$

"secured on the x-axis at $x=0$ and $x=\pi$"
"starts from rest from the int. disp. $f(x)$"
(initial displacement)

BC) $u(0,t)=0$ $u(\pi,t)=0$

IC) $\frac{\partial u}{\partial t}(x,0)=0$ $u(x,0)=f(x)$

$u(x,t)=X(x)T(t)$

$X''(x)T(t) = X(x)T''(t) + 2\beta X(x)T'(t)$

$$\frac{X''(x)}{X(x)} = \frac{T''(t)+2\beta T'(t)}{T(t)}$$

$X''(x) = \text{constant } X(x) = -\lambda^2 X(x)$

$X(x) = \sin(nx)$

$\frac{T''(t)+2\beta T'(t)}{T(t)} = -n^2$ $T''(t)+2\beta T'(t)+n^2T(t)=0$

↑ solve this ODE!

use the characteristic Eq.
$k^2+2\beta k+n^2=0$

$k = \frac{-2\beta \pm\sqrt{4\beta^2-4n^2}}{2} = -\beta \pm \sqrt{\beta^2-n^2}$

$\beta^2 - n^2$ is NEGATIVE, therefore we get the interesting formulas for $T(t)$

$$T(t)\begin{cases} e^{-\beta t}\sin(\sqrt{n^2-\beta^2}\,t) \\ e^{-\beta t}\cos(\sqrt{n^2-\beta^2}\,t) \longleftarrow \frac{\partial u}{\partial t}(x,0)=0 \end{cases}$$