

$$\sum b_n \sin(nx) e^{-\beta t} \cos(\sqrt{n^2 - \beta^2} t)$$

$$b_n = \frac{2}{\pi} \int_0^\pi f(x) \sin(nx) dx$$

"starts from rest from the x-axis at x=0 and x=π"

"starts from rest from the x-axis at x=0 and x=π"

(Inversion technique)

$$0 = (1, \pi) \cup (0, \pi) \cup (0, 0) \cup (0, \pi)$$

$$0 = (0, x) \cup (0, 0) \cup (0, \pi)$$

$$T(x, t) = T(x, 0)$$

$$T(x, t) = T(x, 0) + \text{constant}$$

$$\frac{T'(x, t) + T''(x, t)}{T(x, t)} = \frac{X'(x)}{X(x)}$$

$$X''(x) = -\lambda^2 X(x)$$

$$X(x) = \sin(\lambda x)$$

$$0 = T''(x, t) + T'(x, t) + T(x, t) = 0$$

↑ solve this ODE!

Use the characteristic eq

$$k^2 + \beta k + \lambda^2 = 0$$

$$k = \frac{-\beta \pm \sqrt{\beta^2 - 4\lambda^2}}{2} = -\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} - \lambda^2}$$

NEGATIVE

there are two roots
the interesting one

formulas for T(x, t)

$$0 = (0, x) \frac{\lambda \beta}{\beta^2} \rightarrow \left. \begin{aligned} &e^{-\beta t} \cos(\sqrt{\lambda^2 - \beta^2} t) \\ &e^{-\beta t} \sin(\sqrt{\lambda^2 - \beta^2} t) \end{aligned} \right\} T(x, t)$$