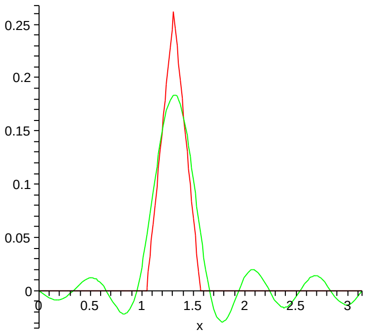
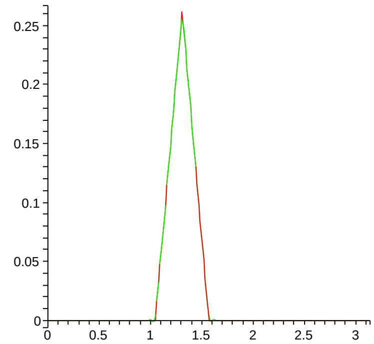
Math 421 **Some vibration examples** **November 29**

I defined a small triangular initial condition for `Maple`:

```
>F:=x->piecewise(x<Pi/3,0,x<Pi/3+Pi/12,x-(Pi/3),x<Pi/2,Pi/3+Pi/6-x,0);
```

This was an initial perturbation of the string. Here is a picture of the initial perturbation, together with the sum of the first 10 terms of its Fourier sine series. To the right is a similar picture, except that what's shown is the sum of the first 100 terms of its Fourier sine series. I can't see any difference between the two curves in the picture on the right.

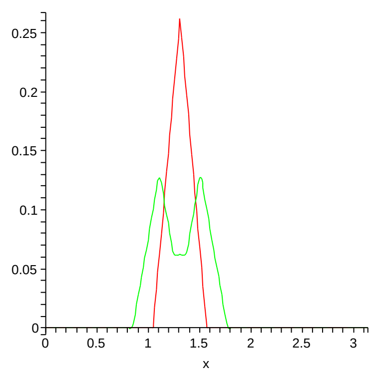




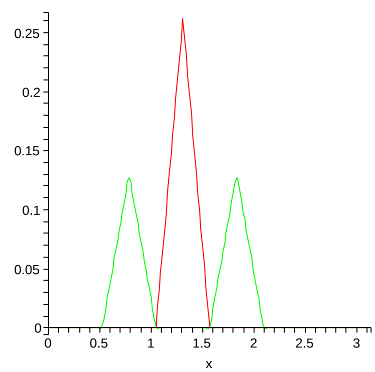
The math equations in back of this: $b_n = \frac{2}{\pi}\int_0^\pi F(x)\sin(nx)\,dx$, so that the partial sum of the Fourier sine series is $Q_N(x) = \sum_{n=1}^N b_n \sin(nx)$.

Now let's "solve" the wave equation with this initial data, and with the boundary conditions corresponding to the ends fastened at 0 and π: so we want $u(x,t)$ satisfying: **PDE** $u_{xx} = u_{tt}$; **BC** $u(0,t) = 0$; $u(\pi,t) = 0$ for all t; **IC** $u(x,0) = F(x)$ and $u_x(x,0) = 0$, both for $0 \le x \le \pi$.

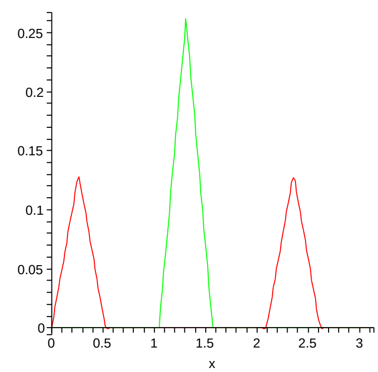
The approximate solution will be $V_N(x) = \sum_{n=1}^N b_n \sin(nx)\cos(nt)$. Here are pictures for various t's:



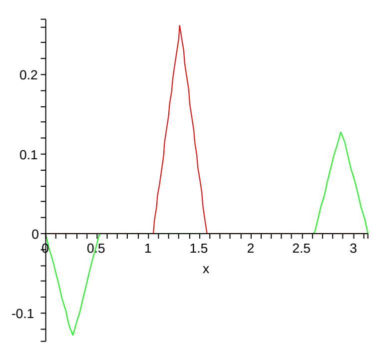
$t = .2$



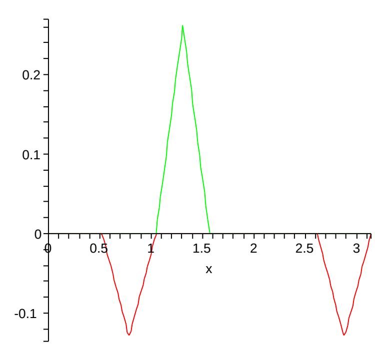
$t = \frac{\pi}{6}$



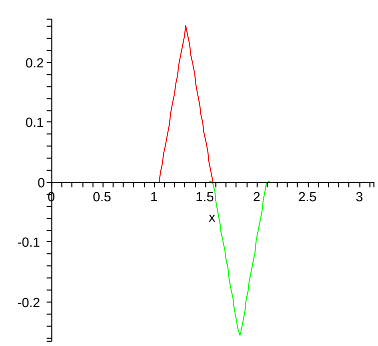
$t = \frac{\pi}{3}$



$t = \frac{\pi}{2}$



$t = \frac{2\pi}{3}$

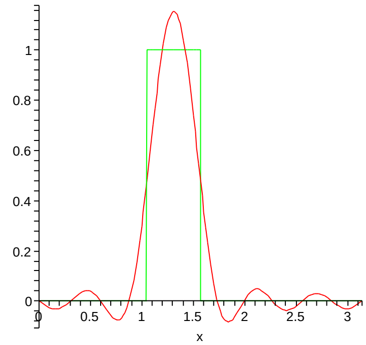


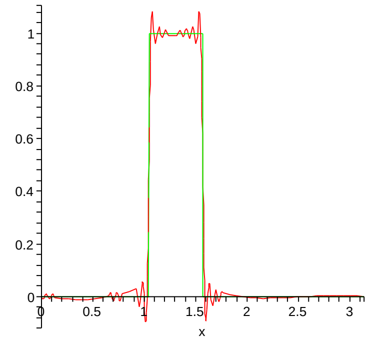
$t = \pi$

Now I'd like to solve an initial velocity problem. Here I'll suppose that the initial velocity of the string is *up* one unit in the interval $[\frac{\pi}{3}, \frac{\pi}{2}]$.

```
>G:=x->piecewise(x<Pi/3,0,x<Pi/2,1,0);
```

And here is a picture of the Fourier sine series, first for $n = 10$ and then for $n = 100$:

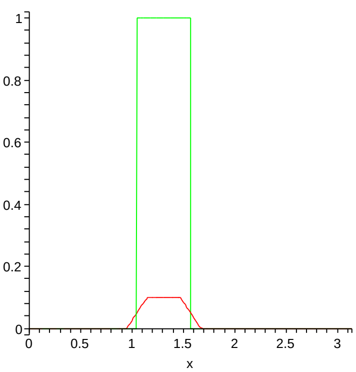




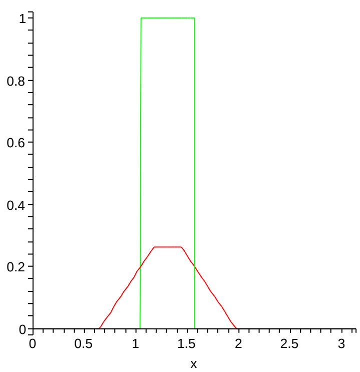
The math equations in back of this: $c_n = \frac{2}{\pi}\int_0^\pi G(x)\sin(nx)\,dx$, so that the partial sum of the Fourier sine series is $Q_N(x) = \sum_{n=1}^N c_n \sin(nx)$.

Now let's "solve" the wave equation with this initial data, and with the boundary conditions corresponding to the ends fastened at 0 and π: so we want $u(x,t)$ satisfying: **PDE** $u_{xx} = u_{tt}$; **BC** $u(0,t) = 0$; $u(\pi,t) = 0$ for all t; **IC** $u(x,0) = 0$ and $u_x(x,0) = G(x)$, both for $0 \le x \le \pi$.

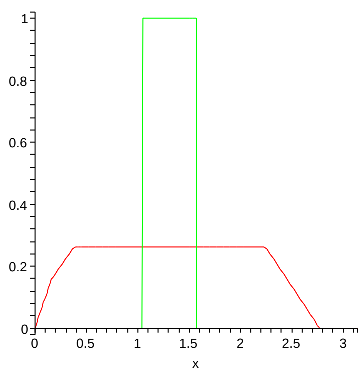
The approximate solution will be $V_N(x) = \sum_{n=1}^N \frac{b_n}{n}\sin(nx)\sin(nt)$. Here are pictures for various t's:



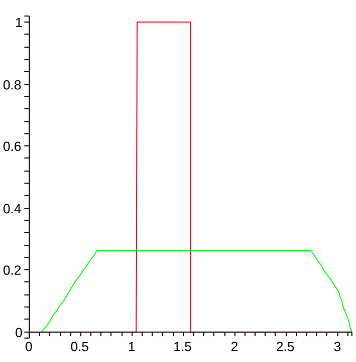
$t = .1$



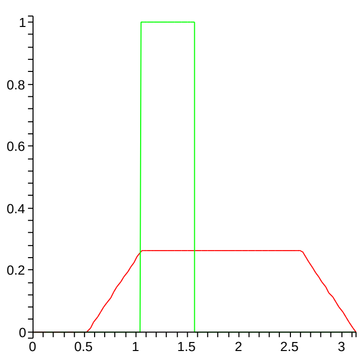
$t = .4$



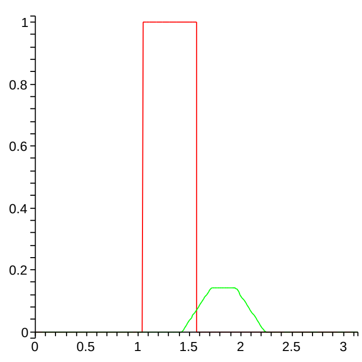
$t = 1.2$



$t = 1.7$



$t = 2.1$



$t = 3$