Math 421: Real matrices

Matrices arising from real applications are rarely random. Sometimes they are *sparse*, meaning they have relatively few non-zero entries for their size (for example, a 2,000 by 2,000 matrix with only about 2,500 non-zero entries). Sometimes matrices have *block form*, with patterns we'd like to use. Some of these uses are correct, and some of them are not.

Earn 5 points towards your second exam grade!

Rules While you may discuss these problems with me and with other students, the work you hand in should be your own. Solutions may be given to me or e-mailed to me. I will accept solutions to these problems until the beginning of class on **Monday, November 14**. Therefore I must receive e-mail solutions by noon of that day.

1. Suppose we have the following matrices:

$$A = \begin{pmatrix} a & b & e & f \\ c & d & g & h \\ 0 & 0 & i & j \\ 0 & 0 & k & l \end{pmatrix} \quad B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}$$

Prove that $\det(A) = \det(B)\det(C)$.

2. Suppose we have the following matrices:

$$A = \begin{pmatrix} a & b & e & f \\ c & d & g & h \\ i & j & m & n \\ k & l & o & p \end{pmatrix} \quad B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad C = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \quad D = \begin{pmatrix} i & j \\ k & l \end{pmatrix} \quad E = \begin{pmatrix} m & n \\ o & p \end{pmatrix}$$

Give an example to show that $\det(A)$ may not be equal to $\det(B)\det(E) - \det(C)\det(D)$.